Multivariate volatility models in financial risk management and portfolio selection

A dissertation presented
by

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to
The Department of Statistics
in partial fulfilment of the requirements
for the degree of
Doctor of Philosophy
in the subject of

Business and Quantitative Methods

Supervisors:
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Universidad Carlos III de Madrid
May, 2010

Abstract

Multivariate volatility modeling is now established as one of the most influential and challenging areas in financial econometrics. Rather than modeling assets separately in a traditional univariate way, research in econometric modeling of volatility has been evolving towards the extension of the univariate framework through the development of multivariate specifications able to model and predict the temporal dependence in the second-order moments of many assets in a portfolio or in different markets taking into account their correlated behavior. Therefore, the use of multivariate volatility models in quantitative risk management has gained increased importance among academics and practitioners concerned with measuring and managing financial risks.

In this thesis we study multivariate volatility models in problems involving quantitative market risk measurement and management. First, we consider the risk measurement problem of forecasting value-at-risk (VaR) using multivariate models vis-à-vis traditional univariate models in problems involving diversified portfolios with a large number of assets. Second, we present a novel active risk management approach based on current regulatory criteria to select optimal portfolio compositions. Finally, I discuss the implications, advantage and caveats of using multivariate volatility models, and propose research lines that can contribute to guide further research in this area.

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Chapter 1

Introduction

Managing risks is undoubtedly one of the most important activities within the financial industry. Banks and other financial institutions are very concerned about all kinds of risks that can potentially damage the financial position of their clients or shareholders. In this sense, two of the most relevant activities in this area are *risk measurement* and *risk management*. The latter can be seen as the calculation, based on historical observations and a given model, of an estimate of the distribution of the change in value of a portfolio consisting of N assets with respective weights $w_1, ..., w_N$. Therefore, as the name suggests, risk measurement is concerned with measuring the risk of a given portfolio of assets. This task is usually performed on a daily basis in most large banks in order to monitor the risk exposure of the aggregate position. Risk management, on the other hand, is the ability to change portfolio compositions so as to earn an adequate return of funds invested, and to maintain a comfortable surplus of assets beyond liabilities. In many practical situations, risk measurement is closely followed by risk management since portfolio managers often have to change portfolio compositions as a response to an increase in the risk of the position.

There are several reasons to justify the importance of risk management. First, from the corporate point of view, risk management can reduce tax costs by reducing the variability in a firm's cash flow and leading to higher expected after-tax profit. Second, risk management makes bankruptcy less likely to occur. In short, it is argued that risk management can create value to the shareholder. Moreover, the first Basel Accord on Banking Supervision in 1998 introduced a regulatory perspective by putting risk management on the front stage of banking activity. This central role was further enforced in the Amendment to Basel I in 1996 and in the Basel II Accord in 2004.

The focus of this thesis is on *market risk*, which is probably the best known type of risk. Market risk can be defined as the risk of a change in the value of a financial position due to changes in the value of the underlying components on which that position depends (McNeil et al. 2005). In particular, market risk can be seen as the risk of losses on positions in equities, interest rate related instruments, currencies and commodities due to adverse movements in market prices. Managing market risk usually depends on the use of econometric models to support strategic decision making, thus giving rise to the so-called area of quantitative risk management (QRM).

One of the most successful approaches in the area of QRM are the multivariate volatility models. Researchers within this area are concerned with the understanding of the dynamics of the second-order moments of asset returns. These models are now established as one of the most influential and challenging approaches in financial econometrics, mainly because some stylized effects in assets returns such as time-varying correlations, contagion, and portfolio diversification are crucial aspects for risk modeling and naturally claim for a multivariate treatment of the volatility dynamics. Therefore, rather than modeling assets separately in a traditional univariate way, the research in econometric modeling of volatility has been evolving towards the extension of the univariate framework through the development of multivariate specifications able to model and predict the temporal dependence in the second-order moments of many assets in a portfolio or in different markets taking into account their correlated behavior.

Multivariate volatility models have been applied in a variety of contexts. First, these models are suitable for studying the relationship among the volatilities across markets. Does a shock on a market increase the volatility on another market, and by how much? Are these shocks symmetric or asymmetric? Are the correlations among financial assets higher during periods of high volatility? Such issues can be studied directly by means of a multivariate model, and raise the question of the specification of the dynamics of covariances and correlations.

The application of multivariate models, however, are not restricted to the study of volatility transmission. Another important research line is in asset pricing models that relate asset returns to factors, which can be either specified a priori or extracted from the data using multivariate techniques such as factor or principal component analysis. Therefore, one can apply a multivariate volatility model to estimate time-varying factor loads; see, for example, Bollerslev et al. (1988) for an early reference. Multivariate models have also been applied in the computation of time-varying hedge ratios. A bivariate volatility model for the spot and future returns can model directly their conditional covariance and time-varying hedge ratios can be computed as the byproduct of estimation and updated by using new observations as they become available (Brooks et al. 2001).

The main area of interest of multivariate volatility models, however, is in market risk modeling. The Basel II Accord explicitly recognizes the role of standard financial risk measures such as value-at-risk (VaR) which financial institutions must adopt and report in order to monitor their short term risk exposure and to compute the amount of economic capital subjected to regulatory control. This context stimulated the appearance of a long list of references in multidisciplinary fields such as finance, economics, mathematics and statistics, concerned with modeling and predicting as accurately as possible financial risk measures such as VaR, thus providing neither too conservative nor insufficient provisions of future possible losses and/or economic capital required for regulatory purposes. It is worth noting that the econometric modeling and forecast of volatility is a crucial point in modeling financial risk; the accuracy of financial risk measures computations depends mainly on the accuracy of volatility modeling and forecast.

Multivariate volatility models also play an important role in portfolio selection problems. In these problems, the two most common parameters that need to be estimated in order to compute the optimal portfolio weights are the vector of expected means and the covariance matrix. The computation of this matrix is usually done without assuming any dynamic dependence in the second-order moments of returns, i.e. they are unconditional or “static” covariance matrices. However, multivariate volatility models offer an alternative way of modeling and forecasting covariance matrices, by assuming a dynamic dependence conditional on the past information for the volatility. Therefore, the plug in estimation procedure could be done by using estimated or forecasted covariance matrices using multivariate volatility models. Recent references in this area are Aguilar and West (2000), Engle and Colacito (2006), Han (2006), Jondeau and Rockinger (2006), and Carvalho and West (2007), among others.

In this thesis, we shed light on the application of multivariate volatility models in market risk modeling. Our interest in is both risk measurement and risk management. First, in the remainder of this chapter, we provide a review of the multivariate (and univariate) volatility models that will be employed in the following chapters. We refer the interested reader to the excellent reviews of multivariate GARCH models in Bauwens et al. (2006) and Silvennoinen and Teräsvirta (2009) and of multivariate stochastic volatility models in Asai et al. (2006). In the second chapter, we consider a risk measurement problem of forecasting portfolio value-at-risk (VaR) with multivariate GARCH models vis-à-vis univariate models. Existing literature has tried to answer this question by analyzing only small portfolios and using a testing framework not appropriate for ranking VaR models. We, on the other hand, provide a more comprehensive look at the problem of portfolio VaR forecasting by using more appropriate statistical tests of comparative predictive ability. Moreover, we compare univariate vs. multivariate VaR models in the context of diversified portfolios containing a large number of assets and also provide evidence based on Monte Carlo experiments. In the third chapter we shift our attention to an active risk management problem and propose a novel optimization problem based on the Basel II capital requirement formula to obtain optimal portfolios with minimum capital requirements subject to a given number of violations over the previous trading year. An illustration involving three data sets of real market data shows that the proposed approach delivers an improved balance between capital requirement levels and the number of VaR exceedances. Finally, in the fourth chapter we provide a summary of the main contribution of the thesis along with lines of future research.

1.1 Multivariate and univariate volatility models

In this section, we describe several alternative procedures to obtain portfolio VaR forecasts using univariate and multivariate GARCH models. Throughout the chapter we focus on the portfolio VaR for a long position in which traders have bought the assets and wish to measure and manage the risk associated to decreases in asset prices.

1.1.1 VaR estimation

Denote by $R_{t+h} = (r_{1,t+h}, \ldots, r_{N,t+h})'$ the vector of h-period returns (between t and $t+h$) of the N assets contained in the portfolio. The portfolio return is given by $r_{p,t+h} = w_t' R_{t+h}$, where w_t is the vector of portfolio weights to be determined at time t. The portfolio VaR at time t for a given holding period h and confidence level α is given by the α-quantile of the conditional distribution of the portfolio return $r_{p,t+h}$. Thus, $\text{VaR}_t(h,\alpha) = F^{-1}_{p,t+h}(\alpha)$, where $F^{-1}_{p,t+h}$ is the inverse of the cumulative distribution function of the portfolio return. Equivalently, the VaR can be defined as:

$$\text{VaR}_t(h,\alpha) = \sup_r F_{p,t+h}(r) \leq \alpha. \tag{1.1}$$

Throughout the paper we focus on the portfolio VaR for a holding period of $h = 1$ day at $\alpha = 1\%$. The latter is the relevant confidence level that banks must adopt in computing their risk exposure. The Basel accord requires the use of VaR estimates for a holding period h of 10 days, but it allows these to be computed from VaR estimates for shorter periods by using the square-root-of-time-rule, that is $\text{VaR}_t(10, \alpha) = \sqrt{10/h}\text{VaR}_t(h, \alpha)$ for some $h < 10$.[1] Therefore, from now on, we eliminate the arguments h and α from the definition of the VaR.

In order to compute the α-quantile conditional distribution of the portfolio return we can consider two alternative conditioning sets. First, we can consider the distribution of portfolio returns conditional on past portfolio returns, i.e. the distribution of $r_{p,t}$ conditional on a fixed linear combination of past asset returns, $w'_{t-h-1}R_{t-h}$. Alternatively, we can consider the distribution of $r_{p,t}$ conditional on the whole vector of past asset returns, R_{t-h}. The former case leads to a univariate model for the portfolio returns while the latter leads to a multivariate analysis. In any of both cases there are two possibilities for the specification of the conditional distribution of portfolio returns. First, the VaR can be estimated without assuming any particular parametric form of this distribution, thus estimating directly its 1% quantile. Alternatively, we can assume a parametric specification by assuming a particular model for the conditional mean and variance and a particular distribution for the standardized returns. Given that in this thesis we focus on high-dimensional portfolios consisting of a large number of assets N, parametric specifications may be more appropriate.

The portfolio return can be represented as

$$r_{p,t+1} = \mu_{p,t+1} + \sigma_{p,t+1}\varepsilon_{p,t+1} \tag{1.2}$$

where $\mu_{p,t+1}$ and $\sigma_{p,t+1}$ are the portfolio conditional mean and standard deviation,

[1] See Bank for International Settlements (2006, paragraph 718(Lxxvi)). Diebold et al. (1998) and Danielsson and Zigrand (2006) discuss the use of the square root rule.

respectively, given by

$$\mu_{p,t+1} = w_t' \mu_{t+1} \tag{1.3}$$

and

$$\sigma^2_{p,t+1} = w_t' H_{t+1} w_t, \tag{1.4}$$

where μ_t is the $N \times 1$ vector of conditional means, $\mu_{t+1} = \mathsf{E}[R_{t+1}|R_1, \ldots, R_t]$, and H_{t+1} is the $N \times N$ conditional covariance matrix, $H_{t+1} = \mathsf{E}[(R_{t+1} - \mu_{t+1})'(R_{t+1} - \mu_{t+1})|R_1, \ldots, R_t]$. The centered and standardized returns ε_{t+1} in (1.2) are independent and identically distributed with mean equal to zero and unit variance, i.e. $\mathsf{E}[\varepsilon_{t+1}] = 0$ and $\mathsf{E}[\varepsilon^2_{t+1}] = 1$ for all t. The portfolio VaR is then given by

$$\mathrm{VaR}_{t+1} = \mu_{p,t+1} + \sigma_{p,t+1} q \tag{1.5}$$

where q is the 1% quantile of the distribution g of $\varepsilon_{p,t+1}$.

It is important to note that the conditional distribution of the portfolio return $w_t' R_{t+1}$ given $\{R_1, ..., R_t\}$ is, in general, unknown. It takes a tractable form when the distribution of returns is closed under linear transformations, i.e. when, for example, all linear combinations of R have the same distribution as the marginal distribution of returns. This is the case of the standardized multivariate Normal and Student t distribution; see Pesaran et al. (2008) and Christoffersen (2009). Therefore, in this thesis we consider two alternative specifications for the conditional distribution: the Gaussian distribution and, in order to take into account the presence of fatter tails, the Student's t distribution[2]. Finally, we note that the portfolio conditional mean may be obtained from linear (vector) autoregressive [(V)AR] models as well as nonlinear models (Carriero et al. 2009; Pesaran et al. 2009; DeMiguel et al. 2010). Alternatively, one can consider a simplifying assumption that the portfolio

[2]Note that, when considering a Student's t distribution (in both multivariate and univariate models), the 1% quantile of the conditional distribution function, q, in (1.5) is given by $q = \sqrt{\frac{v-2}{v}}\tilde{q}$, where $\tilde{q}$ is the 1% quantile of a Student's t distribution with v degrees of freedom; see Pesaran and Pesaran (2007, section 5).

conditional mean is equal to zero, i.e. $\mu_{p,t} = 0$. This assumption is reasonable when dealing with daily data.

1.1.2 Univariate volatility models

As we mentioned above, parametric univariate models for calculating the VaR are based on assuming a particular variance and distribution of portfolio returns given past portfolio returns. We consider that the distribution of $\varepsilon_{p,t}$ can be either a Gaussian or a Student's t distribution with v degrees of freedom. Moreover, we consider two different specifications for the portfolio conditional standard deviation $\sigma_{p,t}$: the GARCH model (Bollerslev 1986) and the asymmetric GJR-GARCH model (Glosten et al. 1993). The GARCH model is given by:

$$\sigma^2_{p,t+1} = \omega + \alpha r^2_{p,t} + \beta\sigma^2_{p,t} \tag{1.6}$$

where $\omega > 0$, $\beta, \alpha \geq 0$ and $\alpha + \beta < 1$ to guarantee the positivity of conditional variances and stationarity of returns. The asymmetric GJR-GARCH model is described as:

$$\sigma^2_{p,t+1} = \omega + \alpha y^2_{p,t} + \beta\sigma^2_{p,t} + \delta I(\varepsilon_t < 0)y^2_{p,t} \tag{1.7}$$

where $I(\cdot)$ is an indicator function that takes value 1 when the argument is true. The restriction to ensure positiveness of $\sigma^2_{p,t+1}$ is $\omega > 0$, $\alpha, \beta, \delta \geq 0$. The model is stationary if $\delta < 2(1 - \alpha - \beta)$; see Hentschel (1995).

We also consider the semiparametric conditional autoregressive VaR model (known as CAViaR) proposed by Engle and Manganelli (2004) which is designed to estimate directly the 1% quantile of the conditional distribution of the returns, which is given by the following expression:

$$VaR_{p,t+1} = (\omega + \alpha y^2_{p,t} + \beta VaR^2_{p,t})^{1/2}. \tag{1.8}$$

The parameters of the univariate GARCH models considered in this work are estimated via quasi maximum likelihood (QML). A review of estimation issues of univariate GARCH models, such as choice of initial values, numerical algorithms and accuracy, is provided by Zivot (2009). It is important to note that even when the normality assumption is inappropriate, maximizing the Gaussian log likelihood results in QML estimates that are consistent and asymptotic normally distributed provided that the conditional mean and variance functions of the GARCH model are correctly specified; see Bollerslev and Wooldridge (1992). Finally, the estimation of the parameters of the CAViaR model is performed by means of regression quantiles; see Engle and Manganelli (2004).

1.1.3 Multivariate volatility models

We consider five different specifications for the conditional covariance H_t: the diagonal VEC model of Bollerslev et al. (1988) and its asymmetric version, the constant conditional correlation (CCC) model of Bollerslev (1990), the dynamic conditional correlation (DCC) model of Engle (2002) and the asymmetric DCC (AsyDCC) model of Cappiello et al. (2006). Moreover, two alternative multivariate distributions for the system of standardized residuals ϵ_t are considered: the Gaussian and the Student's t distribution with v degrees of freedom.

The diagonal VEC(1,1) model (hereafter DVEC(1,1)) of Bollerslev et al. (1988) is given by:

$$H_{t+1} = C + A \odot R_t R_t' + B \odot H_t \tag{1.9}$$

where $\odot$ denotes the Hadamard (elementwise) product, and C, A and B are positive definite squared symmetric matrices. In this model each covariance depends on its own past values and shocks. Besides, the model is covariance-stationary if the eigenvalues of $A + B$ are all less than 1 in modulus. In order to represent stylized facts such as conditional asymmetries, Engle and Sheppard (2008) proposed an asymmetric version of the DVEC(1,1) model, hereafter AsyDVEC(1,1), which is given by:

$$H_{t+1} = C + A \odot R_t R_t' + B \odot H_t + G \odot \eta_t \eta_t' \tag{1.10}$$

where A, B and G are positive definite matrices and $\eta_t = I(Y_t < 0) \odot Y_t$. By taking expectations, the matrix C can be rewritten as $\bar{H} \odot (\iota\iota' - A - B) - \bar{N} \odot G$, where ι is a vector of ones, $\bar{N} = E[\eta_t \eta_t']$ and $\bar{H}$ is the unconditional covariance matrix. A sufficient condition to ensure the positive definiteness of H_{t+1} in the AsyVEC(1,1) model is that $\bar{H} \odot (\iota\iota' - A - B) - \bar{N} \odot G$ and the matrix H_1 are positive definite; see Cappiello et al. (2006) and Engle and Sheppard (2008). Both DVEC(1,1) and Asy-DVEC(1,1) models are highly parameterized. For example, for a 10-asset portfolio the DVEC(1,1) has 75 parameters. Therefore, in this work we only consider these two models as data generating processes (DGPs) in the Monte Carlo simulations in Section 2.4.

Conditional correlation models are currently one of the most promising alternatives to model and forecast conditional covariances. They are based on the decomposition of the conditional covariance matrix into conditional standard deviations and correlations; see Bauwens et al. (2006) and Silvennoinen and Teräsvirta (2009) for reviews and Engle and Sheppard (2001) for comprehensive theoretical and empirical analysis of this class of models. One of their greatest advantages is that they have a smaller number of parameters than traditional multivariate models such as VEC and BEKK models, and therefore can be applied to problems involving a large number of assets. In models of conditional correlations the conditional covariance matrix H_{t+1} is decomposeed as:

$$H_{t+1} = D_{t+1} P_{t+1} D_{t+1} \tag{1.11}$$

where $D_{t+1} = \text{diag}\left(\sqrt{h_{1,t+1}}, \ldots, \sqrt{h_{N,t+1}}\right)$ with diag$(\cdot)$ being the operator that transforms a $N \times 1$ vector into a $N \times N$ diagonal matrix. The conditional variances $h_{j,t+1}$, $j = 1, \ldots, N$, are assumed to follow a standard univariate GARCH(1,1) model. P_{t+1} is a symmetric positive definite conditional correlation matrix with elements

$\rho_{ij,t+1}$, where $\rho_{ii,t+1} = 1$, $i, j = 1, \ldots, N$.

The CCC model of Bollerslev (1990) assumes that the conditional correlation matrix P_t is constant over time, i.e. $P_t = P$ where P is the unconditional correlation matrix of the standardized returns. The CCC model was further extended by Engle (2002)[3] in order to allow time-varying dynamic conditional correlations. In the DCC model the conditional correlation $\rho_{ij,t+1}$ is given by

$$\rho_{ij,t+1} = \frac{q_{ij,t+1}}{\sqrt{q_{ii,t+1} q_{jj,t+1}}} \tag{1.12}$$

where $q_{ij,t+1}$, $i, j = 1, \ldots, N$, are collected into the $N \times N$ matrix Q_{t+1}, which is assumed to follow GARCH-type dynamics,

$$Q_{t+1} = (1 - \alpha - \beta)\,\bar{Q} + \alpha z_t z_t' + \beta Q_t \tag{1.13}$$

where $z_t = (z_{1t}, \ldots, z_{Nt})$ with elements $z_{it} = \varepsilon_{it}/\sqrt{h_{it}}$ being the standardized residuals, $\bar{Q}$ is the $N \times N$ unconditional covariance matrix of z_t and α and β are non-negative scalar parameters satisfying $\alpha + \beta < 1$.

More recently, Cappiello et al. (2006) extended the DCC model to incorporate asymmetric effects in the conditional correlations, yielding the asymmetric DCC (AsyDCC) model. In the AsyDCC model the dynamics of Q_t are now described by:

$$Q_{t+1} = (\bar{Q} - \alpha\bar{Q} - \beta\bar{Q} - \delta\bar{\Gamma}) + \alpha z_t z_t' + \beta Q_t + \delta n_t n_t' \tag{1.14}$$

where $n_t = I\,(z_t < 0) \odot z_t$ and $\bar{\Gamma} = E\,[n_t n_t']$. Cappiello et al. (2006) note that a necessary condition for Q_{t+1} to be positive definite is that $\alpha + \beta + \lambda\delta < 1$, where λ is the maximum eigenvalue of $\bar{Q}^{-1/2}\bar{N}\bar{Q}^{-1/2}$.

When assuming a Gaussian distribution for the errors, we use the two-step procedure proposed by Engle and Sheppard (2001) for the QML estimation of the DCC

[3] An alternative conditional correlation model with time-varying correlation matrices was also proposed by Tse and Tsui (2002).

models considered in this work. Theoretical and empirical properties of this estimation procedure are detailed in Engle and Sheppard (2001) and Sheppard (2003). Some functions available in the Matlab-based toolbox USCD_GARCH were used in the QML estimation of multivariate models[4]. An alternative estimation procedure for time-varying conditional correlation models was also proposed by Engle et al. (2008).

Alternative multivariate models

Widely used by practitioners, the Risk Metrics (RM) approach consists of an exponentially-weighted moving average scheme to model conditional covariances. The model is given by

$$H_{t+1} = (1-\lambda)R_t R_t' + \lambda H_t, \tag{1.15}$$

with the recommended value for the model parameter for daily returns being $\lambda = 0.94$.

Our final approach to model the covariance matrix is the shrinkage estimator of Ledoit and Wolf (2003) (LW). Shrinkage estimators are becoming very popular in the portfolio construction literature due to their ability to reduce the estimation error in large covariance matrices. For instance, Ledoit and Wolf (2003) and Ledoit and Wolf (2004b) report improved results in terms of portfolio performance when the shrinkage estimator is used vis-à-vis traditional estimators such as the sample covariance matrix. In this paper we consider the shrinkage estimator proposed by Ledoit and Wolf (2003), which is defined as an optimally weighted average of the sample covariance matrix and the covariance matrix based on Sharpe (1963) single-index model. The intuition behind this shrinkage estimator is to come up with an optimal convex combination between an unbiased covariance matrix estimator that may be subject to substantial estimation error (i.e. the sample covariance matrix) and another estimator that possibly is biased but has considerably less estimation

[4]The toolbox is available in the link `http://www.kevinsheppard.com/wiki/UCSD_GARCH`.

error (i.e. the covariance matrix from the single factor model). In this model the returns of asset i are described by:

$$r_{it} = a_i + b_i r_{mt} + v_{i,t}, \tag{1.16}$$

where r_{mt} is the market portfolio return.[5] The residuals v_i are assumed to be uncorrelated with market returns and to exhibit no serial correlation. The covariance matrix F of the returns R_t implied by this model is:

$$F = \sigma_m^2 bb' + \Delta, \tag{1.17}$$

where σ_m^2 is the variance of the market returns, b is the vector of slopes or factor loadings, and Δ is a diagonal matrix containing variances of the residuals v_t. The shrinkage estimator of Ledoit and Wolf (denoted by H_{LW}) then is defined as

$$H_{LW} = \psi F + (1 - \psi) S, \tag{1.18}$$

where ψ is the shrinkage intensity and S is the unconditional sample covariance matrix. A closed-form solution for the optimal shrinkage intensity (minimizing the distance between the true and estimated covariance matrices based on the Frobenius norm) is provided by Ledoit and Wolf (2003).[6]

[5] We follow Ledoit and Wolf (2003) and consider that the composition of the market portfolio is given by an equally-weighted combination of the assets belonging to the portfolio under consideration.

[6] Code for computing the optimal shrinkage intensity is available at `http://www.iew.uzh.ch/institute/people/wolf/publications.html`.

Chapter 2

Forecasting portfolio VaR: multivariate vs. univariate models

2.1 Introduction

Market risk management has been receiving increased attention in the last few years due to the importance devoted by the Basel II Accord to the regulation of the financial system. Basel II explicitly recognizes the role of standard financial risk measures, such as VaR, that financial institutions must implement and report in order to monitor their risk exposure and to determine the amount of capital subjected to regulatory control (Berkowitz and O'Brien 2002). Consequently, VaR is now established as one of the most popular risk measures designed to controlling and managing market risk. The Accord also establishes penalties for inadequate models, and consequently, there are incentives to pursue accurate approaches to estimate the VaR. A myriad of models are currently available for modeling the VaR, but no consensus has been reached on which model or method is the best.

The first decision one has to take when trying to predict the VaR of a portfolio is whether to use a multivariate model for the system of asset returns contained in it or, alternatively, assuming a known vector of weights and modeling the univariate time series of portfolio returns. The question that immediately emerges is: Which method is able to provide more accurate VaR forecasts? Each of these two alternatives have several pros and cons. First, the univariate model has to be estimated each time that the vector of weights changes because this yields a different univariate time series of portfolio returns; see Bauwens et al. (2006). This requirement is not necessary when a multivariate model is fitted. Moreover, one can possibly argue that modeling the joint dynamics of the assets contained in the portfolio via a multivariate model can also lead to forecast improvements due to the use of more information. However, as the dimension of the problem increases, the estimation of multivariate models becomes more complicated due to the usually large number of parameters involved; see McAleer (2009). As a consequence, the predictive ability of these models can be compromised. Therefore, the trade-off between estimation difficulties and forecasting performance is not clear at a first glance.

Bauwens et al. (2006) conjecture that, under the current state-of-the-art, it is probably better to adopt univariate models. More recently, Christoffersen (2009) also argues that univariate models are more appropriate if the purpose is risk measurement (e.g. VaR computation) whereas multivariate models are more suitable for risk management (e.g. portfolio selection). Furthermore, most of the existing empirical papers have focused on the analysis of only one class of models, without any comparative analysis among competing approaches. For instance, Engle and Manganelli (2001), Giot and Laurent (2004) and Kuester et al. (2006) analyze VaR forecasting performance among univariate models while Engle (2002), McAleer and da Veiga (2008a) and Chib et al. (2006) provide a similar analysis among multivariate models. In any case, they did not provide a direct comparison of VaR predictive performance among univariate and multivariate volatility models when implemented to the same data set. One exception is the work of Berkowitz and O'Brien (2002), who conclude that a simple univariate model is able to improve the accuracy of portfolio VaR estimates delivered by large US commercial banks. On the other hand, Brooks and Persand (2003) also conclude that there are no gains from using multivariate models, while, more recently, McAleer and da Veiga (2008b) found mixed evidence. However, the empirical analysis of these authors are based on portfolios composed of very few assets (3 and 4, respectively), while in real-world situations, financial institutions are usually faced with much larger portfolios. Furthermore, they compared univariate and multivariate VaRs by using traditional backtesting tests based on coverage/independence criteria (Kupiec 1995; Christoffersen 1998). These tests, though very useful to evaluate the accuracy of a single model, can provide an ambiguous decision about which candidate model is better. Therefore, it is better to use formal statistical tests designed to evaluate the comparative predictive performance among candidate models or, in other words, to compare in a straightforward way the performance of one model versus the other. Finally, Brooks and Persand (2003) and McAleer and da Veiga (2008b) only consider in their empirical analysis multivariate models with constant conditional correlations. There is, however, large evidence that, in practice, conditional correlations move over time; see, for example, Engle (2002), Tse and Tsui (2002) and Cappiello et al. (2006), among many others.

The goal of this chapter is to compare univariate and multivariate GARCH models when implemented to forecast the VaR of large portfolios. The comparison among the alternative models considered in this chapter is done by using the formal statistical tests of superior predictive ability proposed by Giacomini and White (2006). We conduct several Monte Carlo experiments using a very general specification for data generating process (DGP) that include stylized facts such as asymmetric effects. Finally, we also provide empirical evidence by estimating the portfolio VaR of three data sets of real market portfolios containing a large number of assets. We show that even in very large systems, if the sample size is moderately large, it could be worth to model the second order dynamics by fitting multivariate models to predict the VaR of a portfolio.

This chapter is organized as follows. Section 2.3 describes the procedure used to evaluate VaR models. Section 2.2 briefly describes the forecasting models used in this chapter. In Section 2.4 we compare both approaches using simulated data, while Section 2.5 reports results based on real market data. Section 2.6 concludes.

2.2 VaR models

In order to perform the comparison of univariate and multivariate VaR models, we will consider the econometric specifications described in section 1.1. In particular, we will consider as univariate specifications the GARCH, GJR and CAVIAR models. As multivariate specifications, we will consider the asymmetric diagonal VEC (AsyDVEC), CCC, DCC, and AsyDCC models. Finally, in this chapter we do not model first order moments. In order words, we assume that the expected portfolio return is equal to zero, i.e. $\mu_{p,t} = 0$. This assumption allows us to attribute the differences in forecasting performance only to the specification used to model second order moments. Moreover, this assumption is reasonable when dealing with daily data.

2.3 Forecast evaluation of VaR models

Forecast evaluation of VaR models is usually done by means of the backtesting analysis of coverage and independence tests proposed by Kupiec (1995) and Christoffersen (1998). However, if the objective is the comparison among competing models, these tests may not be the best option to provide an unambiguous ranking regarding which candidate model offers superior VaR predictive performance. Instead, it is probably better to use an statistical test to compare in a straightforward way the performance of one model versus the other. In order to achieve this goal, a number of VaR-based comparative predictive ability tests have been proposed; see, for instance, Christoffersen et al. (2001), Giacomini and Komunjer (2005) and, in a more general context of predictive ability, Giacomini and White (2006). In this chapter, we use this last test, known as conditional predictive ability (CPA) test, because it can be applied to interval forecasts and it allows the comparison between nested and nonnested models and among several alternative estimation procedures.

In this chapter, the CPA test is carried out by assuming an asymmetric linear (tick) loss function $\mathcal{L}$ of order α defined as:

$$\mathcal{L}_\alpha\left(e_{t+1}\right) = \left(\alpha - \mathbf{1}\left(e_{t+1} < 0\right)\right) e_{t+1} \tag{2.1}$$

where $e_{t+1} = r_{p,t+1} - \text{VaR}_{t+1}$. As Giacomini and Komunjer (2005) argue, the tick loss function is the implicit loss function whenever the object of interest is a forecast of a particular α-quantile, where $\alpha \in (0, 1)$. Therefore, this function can be considered the relevant loss function for the VaR problem[1]. Moreover, there are at least two important features regarding the use of the tick loss function vis-à-vis traditional backtesting techniques. First, as Lemma 1 in Giacomini and Komunjer (2005) shows, Christoffersen's (1998) correct conditional coverage criterion can be alternatively expressed as $E_t[(\alpha - \mathbf{1}(e_{t+1} < 0))\, e_{t+1}] = 0$. Thus, "*correct conditional coverage condition is equivalent to requiring optimality of an interval forecast with respect to the tick loss function*" (Giacomini and Komunjer 2005, p.419). Second, the tick loss function takes into account the magnitude or the *implicit cost* associated to VaR forecasting errors, in this case e_{t+1}. Since VaR estimates are frequently used to help strategic financial decision-making process and to manage market risk, VaR forecasting errors can imply financial distresses such as misestimation of capital subjected to regulatory control. Therefore, finding the model that minimizes the relevant cost function is an intuitive, appealing criterion to compare predictive ability.

Under the null hypothesis of equal predictive ability, the loss difference between

[1] To see how the tick loss function works in practice, consider a simple example involving two different VaR models. Suppose that the portfolio return in day t is -4% and that the VaR in day t (forecasted in $t-1$) obtained from the two models is -2% and -6%, respectively. Obviously, for the first model there is a VaR violation whereas for the second there is not. For the first model, the value of the tick loss function in (2.1) is $(0.01 - 1)(-2) \cong 2$ whereas for the second model the value is $(0.01 - 0)2 = 0.02$. (Recall that, since we are considering only a long position in the portfolio, the VaR will be always a negative number). Therefore, according to the tick loss function, a model is more penalized when a VaR violation is observed. Moreover, the greater is the magnitude of the violation the greater is the penalization.

two models follows a martingale difference sequence. A Wald-type test of the following form is conducted:

$$CPA = T\left(T^{-1}\sum_{t=1}^{T-1}\mathcal{I}_t\mathcal{LD}_{t+1}\right)'\hat{\Omega}^{-1}\left(T^{-1}\sum_{t=1}^{T-1}\mathcal{I}_t\mathcal{LD}_{t+1}\right) \tag{2.2}$$

where T is the sample size, $\mathcal{LD}$ is the loss difference between the two models, $\mathcal{I}$ is the set of instruments that help predicting differences in forecast performance between the two models, and $\hat{\Omega}$ is a matrix that consistently estimate the variance of $\mathcal{I}_t\mathcal{LD}_{t+1}$. Following Giacomini and White (2006) we assume $\mathcal{I}_t = (1, \mathcal{LD}_t)$. The null hypothesis of equal predictive ability is rejected when $CPA > \chi^2_{T,1-\alpha}$. Giacomini and White (2006) note that the statistic CPA can be alternatively computed as TR^2, where R^2 is the uncentered squared multiple correlation coefficient for the artificial regression of the constant unity on $(1, \mathcal{LD}_t)$.

2.4 Monte Carlo evidence

In this Section we perform Monte Carlo experiments in order to compare the in-sample and out-of-sample performance of multivariate versus univariate models. Our Monte Carlo experiment consists in the following steps

1. Simulate a multivariate system with 10 assets and sample size of T=5,000 observations. The DGP used is the AsyDVEC(1,1) model in (1.10) with Gaussian errors. We chose this model as DGP because it is a very general specification for the dynamics of asset returns that takes into account time-varying second moments and also asymmetric effects. The parametrization of the simulated model is shown in Table 2.1. As we comment next, the results are not affected by the choice of the DGP and the parametrization. Furthermore, it is worth noting that since the AsyDVEC is the true DGP, we do not use this model to estimate the parameters using the simulated data since this would give an unfair advantage to this approach in comparison to the other competing mod-

els. Finally, we focus on the case of a long position in an equally-weighted portfolio. This portfolio composition has been been extensively used in the empirical literature; see, for instance, DeMiguel et al. (2009);

2. Use the first 2,500 observations of the simulated data to estimate each of the multivariate and univariate models described in Section 1.1, except the DVEC and AsyDVEC models;

3. For each estimated model, obtain in-sample one-step-ahead forecasts for the portfolio VaR using the first 2,500 observations;

4. For each estimated model, use the remaining 2,500 observations to provide one-step-ahead out-of-sample forecasts. For each estimated model, use the remaining 2,500 observations to provide one-step-ahead out-of-sample forecasts. These forecasts are nonadaptative, i.e. the parameters estimated using the first half of the sample were kept fixed in the second half of the sample. We also considered the case in which the parameters of all models are re-estimated in a rolling window basis. This procedure, however, is very time consuming. Furthermore, the results are very similar to those of fixed window estimates.

Steps 1 to 4 are repeated 100 times. In each Monte Carlo simulation we compute the average mean squared error (MSE) of the estimated portfolio VaR with respect to the true portfolio VaR obtained from the simulated data, and the CPA test for the pairwise comparisons between multivariate and univariate models. Therefore, after the last 100th Monte Carlo simulation, we have a 100×1 vector of average MSEs and a 100×1 vector of CPA statistics for each pairwise comparison among multivariate and univariate models. This analysis allows us to evaluate the moments of the distribution of the MSEs and also the number of times a model outperformed the other according to the CPA test.

Figure 2.1 plots the Monte Carlo in-sample and out-of-sample distribution of the MSE of the differences between the estimated and true portfolio VaR. Obviously, a higher MSE can be interpreted as a high deviation from the true portfolio VaR, indicating a poor performance. Multivariate models systematically achieved lower MSEs for both in-sample and out-of-sample periods. This can be understood as an indication that multivariate models can perform better than univariate models for the problem of portfolio VaR forecasting. The worse in-sample and out-of-sample performance among univariate models was achieved by the GJR model.

The Monte Carlo results of the Giacomini and White (2006) CPA test are summarized in Table 2.2, which reports the number of times in which each multivariate model outperformed each univariate model; the threshold confidence level is 10%. For instance, the out-of-sample comparison of the DCC versus the GARCH model indicated that the multivariate model was preferred 40 times, the univariate model was preferred in 3 times and in 57 times the performance of the two models was statistically equal. The results in Table 2.2 show that when comparing univariate and multivariate models within-sample, in approx. 80% of the times, both approaches are similar. However, among the cases in which one of the models is better than the other, the selected model is multivariate. One exception to this conclusion is when the multivariate models are compared to the univariate GJR model. In this case, the multivariate and univariate GJR model are only indifferent in around 51% of the simulated systems, with the multivariate model being preferred in 45% of the systems. Therefore, by looking at the within-sample performance of the models, it may seem that with the exception of the GJR model, the advantages of the multivariate versus univariate models is very mild. However, the advantage of the multivariate models appear much clearly when looking at the out-of-sample results. In this case, the multivariate models outperform the multivariate GJR model in nearly all of the simulated systems. For the rest of multivariate models, approximately half of the times, the multivariate and univariate models are similar. When one of the models is selected, with few exceptions, the selected model is the multivariate. For instance, the DCC model was chosen in 40 of the simulated systems in comparison to the GARCH model, which was chosen only in 3 of them. In comparison to the CAVIAR model, the DCC model was chosen in 49 of the simulated systems whereas the CAVIAR model was chosen in 2 of them.

Table 2.2 also reports the pairwise comparisons among only multivariate and among only univariate models. The comparison among multivariate models indicates that dynamic conditional correlation models are preferred to the constant conditional correlation model nearly the same number of times as the CCC model is preferred to the DCC. In any case, it is clear that both models are preferred to the AsyDCC model. The comparison among univariate models delivered mixed results. The CAVIAR and GARCH models performed similarly in approx. 50% of the times, but the latter was selected more often than the former in the within-sample period, while the opposite result was observed in the out-of-sample period. In both periods, however, these two univariate models outperform the GJR model.

The Monte Carlo results indicate that multivariate models perform better than univariate models when applied to the problem of portfolio VaR forecasting. However, one can possibly argue that these results might be driven by a specific choice of the parameters and the choice of DGP specification. In order to rule out this possibility, we have performed the same analysis with different parameter sets, and also with different specifications for the DGP. The results are very similar to those reported here, and are not reported to save space.

2.5 Empirical evaluation with real market data

In this Section we compare multivariate and univariate models by using real market data to forecast one-day-ahead VaR for a long position in equally-weighted portfolios. We are now interested in evaluating the performance of each model under more realistic situations, i.e. when the portfolio has a very large number of assets and is diversified, including not only stocks but also bonds, commodities and foreign currencies. This is usually the case in most financial institutions.

We consider portfolios constructed from three sets of different types of assets. The first set consists of 30 futures contracts on equity indices (S&P500, NASDAQ, DJIA, Canada 60, FTSE, CAC, DAX, IBEX, MIB, Nikkei, Hang Seng, SGX, Bovespa, IPC), commodities (gold, silver, wheat, and crude), currencies (euro, British pound, Japanese yen, Canadian dollar, Swiss franc, Australian dollar, Mexican peso and Brazilian real) and 10-year government bonds (US, UK, Germany, and Japan). For each contract we measure returns in dollars, and implement appropriate adjustments for roll-overs from one futures contract to the next. The second set of assets comprises 48 US industry portfolios[2]. The third set of assets consists of all stocks that belonged to the S&P100 index during the complete sample period. This yields a total of 81 stocks. For all three sets of assets we obtain daily observations from March 1, 2000 until July 31, 2008. Returns are computed as the differences in log prices. Table 2.3 shows the number of observations, the mean, standard deviation, skewness, and kurtosis for each data set. The statistics are based on an equally-weighted portfolio and the sample is divided into in- and out-of-sample observations. The fist $T - 500$ observations correspond to the in-sample period whereas the remaining 500 observations correspond to the out-of-sample period, where T is the length of each data set.

2.5.1 VaR estimation

For each of the three data sets analyzed, the within-sample observations are used to estimate the parameters and the remaining 500 observations are used to obtain out-of-sample forecasts. We obtain out-of-sample forecasts using a fixed estimation window similar as in the Monte Carlo simulations. In unreported results, we also analyzed the case in which forecasts were obtained via rolling windows, with similar conclusions. It is worth noting, however, that the computational effort of ob-

[2]This data set was obtained from the web page of Kenneth French (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/`)

taining rolling windows forecasts is extremely high, since all multivariate models need to be re-estimated 500 times.

The following multivariate and univariate models are considered: DCC, AsyDCC, CCC, GARCH, GJR, and the semiparametric CAViaR model, yielding a total of three univariate and three multivariate models. For each of the parametric models, we fit both the model with Gaussian and with Student's t errors. Tables 2.4, 2.5 and 2.6 report the estimated parameters of all six models for each of the three data sets considered in this chapter, respectively. The parameter estimates are similar to those found in previous works by other authors. For instance, the values of the DCC and AsyDCC parameters are similar to those reported in Cappiello et al. (2006), whereas the value of the CAViaR parameters are similar to those reported in Engle and Manganelli (2004). Similar as in Engle and Sheppard (2001), we found that the estimated news parameter, $\hat{\alpha}$, in the DCC models are small ($\hat{\alpha} < .01$), although significant. It is also worth highlighting some interested findings revealed in the estimation results. The estimated number of degrees of freedom v differ across Student's t distributed models. In general, univariate models tend to estimate a larger value for the degrees of freedom in comparison to their multivariate counterparts. The number of degrees of freedom in the multivariate Student's t models ranged from 10 to 19, similar to those reported in Pesaran and Pesaran (2007). Furthermore, the parameter δ associated to the asymmetric term in both univariate and multivariate models is significant in the majority of the cases. In particular, the asymmetric term in the AsyDCC model has a higher significance when a Student's t distribution is used to fit the model, suggesting that the asymmetric multivariate model is better fitted when fat-tailedness is taken into account. Finally, the estimated $\hat{\alpha}$ parameters in the GJR model is not significant in any of the three portfolios considered.

Figures 2.2 to 2.4 plot the out-of-sample VaRs predicted by each of the models for the three data sets according to each model. The evolution of the VaR estimates obtained by multivariate models tended to be smoother in comparison to multivariate models. In general, the VaR estimations obtained by all models have a similar evolution. However, albeit useful, this visual inspection does not allow us to draw an appropriate statistical evaluation of the accuracy of multivariate and univariate models. Therefore, we now proceed to the analysis of the Giacomini-White CPA test as described in Subsection 2.3.

Table 2.7 reports the results of the Giacomini-White CPA test for the 48 industry portfolio. In this Table, after each CPA coefficient, a left (up) arrow means that the model in the row outperforms (underperforms) the model in the column. p-values appear bellow each CPA coefficient. The upper and middle panels show the result for the Normally and Student's t distributed models, respectively, whereas the lower panel shows a comparison among them. The results for the Normally distributed models indicate that multivariate models outperformed univariate models. The best performance was achieved by the DCC model. The results for the Student's t distributed models also indicate that multivariate models performed better, and that the best performance was achieved by the CCC-t model. Finally, the lower panel in Table 2.7 indicates that Normally distributed multivariate models performed better than Student's t distributed univariate models. Overall, the best performance was achieved by the DCC model with Gaussian errors.

Table 2.8 reports the results of the Giacomini-White CPA test for the 30-asset global portfolio. The results for Normally distributed models show that multivariate models performed better than univariate models, and the best performing models are the DCC and CCC models. For the case of Student's t distributed models, multivariate models also outperformed univariate counterparts, and the best performance was achieved by the CCC-t model. Furthermore, similar as in the previous case, the lower panel results indicates that the overall best performance was also achieved by the DCC with Gaussian errors and CCC-t models.

Table 2.9 reports the results of the Giacomini-White CPA test for the S&P100 stocks. The results are very similar to those reported in Table 2.8: The Normally distributed multivariate models outperformed their univariate counterparts and among Student's t distributed models, the best performance was achieved by the CCC-t model. Finally, the comparison among the Normally and Student's t distributed models indicated that, similar as in the 30-asset global portfolio, the best overall performance was achieved by the DCC model with Gaussian errors.

Finally, it is worth noting that although the asymmetric term in the AsyDCC model is significant in the majority of the cases, this model is usually outperformed by the (symmetric) DCC model. This result is in line with our previous findings obtained via Monte Carlo simulations in Section 2.4. Moreover, we found that among Normally distributed models dynamic conditional correlations models are preferred, whereas among Student's t distributed model the preferred model is the one with constant conditional correlations. In any of the two cases, the best performing model is always multivariate.

2.5.2 Capital requirement analysis

Our aim in this Subsection is to compare, or relate, the statistical results previously obtained with the requirements established by the current regulatory framework set by the Basel II Accord. Under the framework of Basel II, the VaR estimates of the banks must be reported to the domestic regulatory authority. These estimates are used to compute the amount of regulatory capital requirements in order to control and monitor financial institutions' market risk exposure and to act as a cushion for adverse market conditions.

The empirical evidence presented by Berkowitz and O'Brien (2002) and Pérignon et al. (2008) show that banks systematically overestimate their VaR, which leads to an excessive amount of regulatory capital. Pérignon et al. (2008) conjecture that the causes of this overinflated VaR can be due to difficulties in aggregating the VaR across different business lines, or because banks don't want neither to put their reputation at risk nor to attract attention (internally and externally). In any of the cases, there is a cost of misestimating the VaR. As Pérignon et al. (2008) argue, one consequence of the exaggeration of banks' own level of risk is that they appear more risky than they actually are. Therefore, pursing models that deliver accurate estimates of this capital can lead to an increase in efficiency and in the accuracy of risk assessments made by investors.

Basel II allows banks to use internal models to obtain their VaR estimates. However, as McAleer and da Veiga (2008b) point out, if this is the case, the banks have to demonstrate that their models are accurate. This is done by means of a backtesting analysis based on the number of VaR violations, i.e. the number of times in which losses exceeded the estimated VaR. The Accord also establishes penalties for bad models in terms of a multiplicative factor k, which is never lower than 3 and it is based on the VaR estimates over the last 250 business days. The penalty zones are described in Table 2.10. The amount of capital charge is thus obtained by the following formula:

$$\text{Capital Requirement}_t = \max\left\{VaR_{t-1}, (3+k)\cdot\overline{VaR}_{60}\right\} \tag{2.3}$$

where $\overline{VaR}_{60}$ is the average VaR over the last 60 business days. Finally, we note that since we are assuming in a long position in the portfolio, the VaR should be represented as a negative number. For the purposes for the computation of capital requirement, however, we assume that the VaR estimate in (2.3) is transformed into a positive number.

It is worth noting that, from a financial institution's point of view, it is desired to pursue a VaR model that yields minimum capital requirements, since this amount of regulatory of capital has an opportunity cost and could be employed in profitable activities. However, given the characteristics of the capital requirement in (2.3), lower levels of capital requirements could be achieved by adopting a VaR model that delivers a high number of violations, which is definitely not a desired outcome. Therefore, there is an important trade-off between capital requirements and number of VaR violations that should be taken into account when evaluating a set of models according to the Basel II criterion.

Table 2.11 reports the mean daily capital requirements (MDCR) and the number of VaR violations for the out-of-sample period. To facilitate the analysis, the number of VaR violations is reported separately for the first and for the second half of the out-of-sample period, i.e. the number of violations is based on 250 observations, which is equivalent to one trading year. Note that, according to the Basel II accord, this is the time period required to evaluate the number of VaR violations.

A result that immediately emerges from the Table 2.11 is that multivariate models delivered lower MDCC in comparison to univariate models in the three data sets considered in this chapter. For the industry portfolio the model that delivered lower MDCC is the DCC-t, whereas for the global portfolio and for the S&P100 stocks the model is the CCC and CCC-t, respectively. The superior performance of multivariate models in terms of MDCC coincides with our previous backtesting analysis based on the Giacomini and White (2006) CPA test. Another important result is that the number of VaR violations is higher in the second half of the out-of-sample period, in comparison to the first half. This is a reflection of a higher volatility clustering in this period (as can be seen in Figures 2.2, 2.3, and 2.4), thus increasing the occurrences of VaR exceptions.

2.6 Concluding remarks

Obtaining accurate risk measures can be seen as the most important objective of a VaR model. This chapter addressed the question of whether multivariate or univariate models are most appropriate for the problem of portfolio VaR forecasting. We compare both types of models in the context of large and diversified portfolios as those are usually encountered in practice. We also consider complex dynamics of variances and covariances with asymmetries and dynamic correlations. Finally, the models are compared by using more appropriate statistics than those used in previous works. The results of comparative predictive performance for one-step-ahead portfolio VaR obtained with both Monte Carlo simulations and with real market data indicate that multivariate GARCH models outperformed competing univariate models on an out-of-sample basis. Furthermore, the results based on the backtesting analysis established by the Basel II Accord indicate that multivariate models delivered lower levels of daily capital requirements in comparison to univariate models. Considering that previous empirical evidence show that banks systematically overestimate their VaR and the amount of regulatory capital, we conclude that the use of multivariate models can improve the estimation of capital requirements, thus attenuating the costs associated to the overestimation of regulatory capital.

Table 2.1: Parametrization of the simulated AsyDVEC(1,1) model with 10 assets

					C					
	1	2	3	4	5	6	7	8	9	10
1	4.121									
2	0.983	2.726								
3	0.769	-0.836	4.036							
4	1.426	0.011	-1.954	3.713						
5	0.189	0.640	-0.336	-0.721	4.529					
6	0.497	0.616	0.838	-0.279	1.139	2.158				
7	-0.226	0.227	0.886	-2.511	1.312	0.739	3.160			
8	-1.156	-0.439	-0.780	-0.518	-0.158	-1.213	-0.330	1.604		
9	1.307	0.537	0.978	-0.567	-1.060	1.154	0.936	-0.539	3.682	
10	0.607	-0.266	0.924	0.876	-0.179	0.607	-0.656	-0.376	0.216	2.790
					A					
	1	2	3	4	5	6	7	8	9	10
1	0.114									
2	0.054	0.104								
3	-0.026	0.001	0.132							
4	-0.033	-0.070	0.014	0.148						
5	0.007	0.039	-0.010	0.011	0.126					
6	-0.043	-0.003	0.102	-0.019	-0.016	0.133				
7	-0.054	0.003	-0.016	-0.019	0.031	0.030	0.100			
8	-0.009	-0.027	-0.017	0.074	0.005	-0.010	-0.003	0.085		
9	0.043	-0.014	-0.023	0.025	-0.003	-0.039	-0.045	0.010	0.093	
10	-0.043	-0.064	0.045	-0.026	-0.022	0.038	-0.009	-0.069	0.004	0.186

					B					
	1	2	3	4	5	6	7	8	9	10
1	0.757									
2	0.748	0.742								
3	0.744	0.736	0.734							
4	0.762	0.755	0.750	0.772						
5	0.775	0.768	0.762	0.782	0.796					
6	0.766	0.760	0.755	0.774	0.787	0.781				
7	0.757	0.749	0.744	0.766	0.776	0.769	0.761			
8	0.740	0.733	0.728	0.747	0.759	0.752	0.741	0.728		
9	0.771	0.764	0.760	0.779	0.791	0.783	0.773	0.755	0.789	
10	0.756	0.750	0.744	0.764	0.775	0.767	0.758	0.742	0.772	0.759

					G					
	1	2	3	4	5	6	7	8	9	10
1	5.6×10^{-5}									
2	4.3×10^{-5}	3.9×10^{-5}								
3	4.2×10^{-5}	3.5×10^{-5}	3.7×10^{-5}							
4	4.8×10^{-5}	4.0×10^{-5}	3.8×10^{-5}	4.5×10^{-5}						
5	4.8×10^{-5}	3.8×10^{-5}	3.8×10^{-5}	4.4×10^{-5}	4.5×10^{-5}					
6	4.3×10^{-5}	3.6×10^{-5}	3.8×10^{-5}	4.0×10^{-5}	4.0×10^{-5}	4.2×10^{-5}				
7	4.7×10^{-5}	3.8×10^{-5}	3.6×10^{-5}	4.1×10^{-5}	4.0×10^{-5}	3.7×10^{-5}	4.2×10^{-5}			
8	4.5×10^{-5}	4.0×10^{-5}	4.0×10^{-5}	4.3×10^{-5}	4.2×10^{-5}	4.3×10^{-5}	4.0×10^{-5}	4.7×10^{-5}		
9	4.7×10^{-5}	3.9×10^{-5}	3.7×10^{-5}	4.4×10^{-5}	4.3×10^{-5}	3.9×10^{-5}	4.0×10^{-5}	4.2×10^{-5}	4.3×10^{-5}	
10	4.8×10^{-5}	3.9×10^{-5}	3.7×10^{-5}	4.2×10^{-5}	4.0×10^{-5}	3.8×10^{-5}	4.2×10^{-5}	4.2×10^{-5}	4.1×10^{-5}	4.3×10^{-5}

Table 2.2: Monte Carlo results on the Giacomini and White (2006) CPA test for the comparison among multivariate and univariate models
The Table reports the number of times (over 100 Monte Carlo simulations) in which multivariate and univariate models outperform each other according to the CPA test.

Multivariate versus Univariate	Multivariate preferred	Univariate Preferred	Indifferent
In Sample			
DCC versus GARCH	14	2	84
DCC versus GJR	45	4	51
DCC versus CAVIAR	14	5	81
AsyDCC versus GARCH	14	3	83
AsyDCC versus GJR	44	5	51
AsyDCC versus CAVIAR	13	4	83
CCC versus GARCH	14	2	84
CCC versus GJR	45	4	51
CCC versus CAVIAR	14	3	83
Out Sample			
DCC versus GARCH	40	3	57
DCC versus GJR	99	0	1
DCC versus CAVIAR	49	2	44
AsyDCC versus GARCH	42	3	55
AsyDCC versus GJR	99	0	1
AsyDCC versus CAVIAR	48	3	44
CCC versus GARCH	40	3	57
CCC versus GJR	100	0	0
CCC versus CAVIAR	49	2	44

Only multivariate	Model 1 preferred	Model 2 preferred	Indifferent
In Sample			
DCC versus AsyDCC	58	29	13
DCC versus CCC	38	34	28
AsyDCC versus CCC	34	49	17
Out Sample			
DCC versus AsyDCC	48	37	15
DCC versus CCC	38	35	27
AsyDCC versus CCC	28	54	18

Only univariate	Model 1 preferred	Model 2 preferred	Indifferent
In Sample			
GARCH versus GJR	43	15	42
GARCH versus CAVIAR	0	45	55
GJR versus CAVIAR	2	70	28
Out Sample			
GARCH versus GJR	100	0	0
GARCH versus CAVIAR	34	21	45
GJR versus CAVIAR	4	90	6

Table 2.3: Descriptive statistics for the three data sets considered in this chapter

	Number of assets	Number of obs.	Mean ($\times$ 100)	Std. dev.	Kurtosis	Skewness
Industry portfolio	48					
In sample		1,617	0.087	0.866	4.582	-0.235
Out sample		500	0.014	0.993	3.772	-0.327
S&P100 stocks	81					
In sample		1,656	0.013	1.045	5.983	0.013
Out sample		500	0.007	0.997	4.688	-0.331
Global portfolio	30					
In sample		1,694	0.020	0.549	4.088	-0.084
Out sample		500	0.047	0.595	4.496	-0.407

Note: descriptive statistics are based on an equally-weighted portfolio.

Table 2.4: Estimated parameters with corresponding asymptotic standard errors in parenthesis. Data set: industry portfolios

	ω	α	β	δ	v
DCC		0.0046	0.9482		
		(0.0007)	(0.0112)		
DCC-t		0.0053	0.9333		18.8528
		(0.0006)	(0.0109)		(1.1192)
AsyDCC		0.0032	0.9510	0.0045	
		(0.0005)	(0.0093)	(0.0017)	
AsyDCC-t		0.0037	0.9373	0.0054	19.0262
		(0.0005)	(0.0086)	(0.0009)	(1.1678)
GARCH	0.0370	0.1098	0.8408		
	(0.0107)	(0.0179)	(0.0262)		
GARCH-t	0.0366	0.1069	0.8438		69.0487
	(0.0108)	(0.0181)	(0.0265)		(64.0360)
GJR	0.0395	0.0170	0.8633	0.1464	
	(0.0085)	(0.0118)	(0.0218)	(0.0245)	
GJR-t	0.0395	0.0169	0.8635	0.1462	531.3362
	(0.0085)	(0.0116)	(0.0216)	(0.0245)	(217.0499)
CAVIAR	0.2086	0.8018	0.7756		
	(0.1672)	(0.0602)	(0.3360)		

Table 2.5: Estimated parameters with corresponding asymptotic standard errors in parenthesis. Data set: global portfolio

	ω	α	β	δ	v
DCC		0.0080	0.9581		
		(0.0016)	(0.0145)		
DCC-t		0.0080	0.9615		10.085
		(0.0013)	(0.0109)		(0.7691)
AsyDCC		0.0074	0.9584	0.0018	
		(0.0016)	(0.0141)	(0.0016)	
AsyDCC-t		0.0072	0.9594	0.0036	10.211
		(0.0011)	(0.0111)	(0.0012)	(0.7914)
GARCH	0.0114	0.0652	0.8970		
	(0.0041)	(0.0124)	(0.0194)		
GARCH-t	0.0100	0.0626	0.9047		12.4795
	(0.0036)	(0.0120)	(0.0183)		(3.3764)
GJR	0.0131	0.0039	0.9086	0.0894	
	(0.0036)	(0.0121)	(0.0182)	(0.0199)	
GJR-t	0.0126	0.0046	0.9095	0.0896	15.586
	(0.0035)	(0.0109)	(0.0191)	(0.0209)	(5.2779)
CAVIAR	0.032	0.8661	0.7026		
	(0.0527)	(0.0408)	(0.8170)		

Table 2.6: Estimated parameters with corresponding asymptotic standard errors in parenthesis. Data set: S&P100

	ω	α	β	δ	v
DCC		0.0020	0.8869		
		(0.0007)	(0.0480)		
DCC-t		0.0044	0.7403		13.341
		(0.001)	(0.069)		(0.5680)
AsyDCC		0.0020	0.8869	0.000	
		(0.0006)	(0.0507)	(0.0011)	
AsyDCC-t		0.0039	0.7538	0.001	13.340
		(0.001)	(0.079)	(0.001)	(0.5706)
GARCH	0.0120	0.0802	0.908		
	(0.0043)	(0.0129)	(0.0142)		
GARCH-t	0.0120	0.0754	0.9123		16.275
	(0.0046)	(0.0137)	(0.0153)		(5.5639)
GJR	0.0108	0.0012	0.9271	0.1224	
	(0.0031)	(0.0087)	(0.0120)	(0.0192)	
GJR-t	0.0117	0.0000	0.9257	0.1251	19.160
	(0.0034)	(0.0052)	(0.0111)	(0.0200)	(7.0425)
CAVIAR	0.0922	0.9041	0.4983		
	(0.0496)	(0.0124)	(0.2253)		

Table 2.7: Giacomini and White (2006) CPA test results. Data set: industry portfolios.

The Table shows test statistics of Giacomini and White (2006) conditional predictive ability test (with p-values). A left (up) arrow means that the model in the row outperforms (underperforms) the model in the column.

Gaussian models	AsyDCC	CCC	GARCH	GJR	CAViaR
DCC	**16.222**$^{\leftarrow}$	**17.324**$^{\leftarrow}$	**15.810**$^{\leftarrow}$	**18.245**$^{\leftarrow}$	**13.433**$^{\leftarrow}$
	0.000	0.000	0.000	0.000	0.001
AsyDCC		**16.289**$^{\leftarrow}$	**15.860**$^{\leftarrow}$	**17.926**$^{\leftarrow}$	**13.396**$^{\leftarrow}$
		0.000	0.000	0.000	0.001
CCC			**17.649**$^{\leftarrow}$	**15.918**$^{\leftarrow}$	**14.165**$^{\leftarrow}$
			0.000	0.000	0.001
GARCH				**27.875**$^{\uparrow}$	**9.487**$^{\leftarrow}$
				0.000	0.009
GJR					**29.676**$^{\leftarrow}$
					0.000
Student's t models	AsyDCC-t	CCC-t	GARCH-t	GJR-t	CAVIAR
DCC-t	3.264$^{\uparrow}$	**18.511**$^{\uparrow}$	**16.647**$^{\leftarrow}$	**13.592**$^{\leftarrow}$	**11.572**$^{\leftarrow}$
	0.196	0.000	0.000	0.001	0.003
AsyDCC-t		**15.255**$^{\uparrow}$	**16.857**$^{\leftarrow}$	**13.757**$^{\leftarrow}$	**11.628**$^{\leftarrow}$
		0.000	0.000	0.001	0.003
CCC-t			**17.812**$^{\leftarrow}$	**16.529**$^{\leftarrow}$	**14.137**$^{\leftarrow}$
			0.000	0.000	0.001
GARCH-t				**31.700**$^{\uparrow}$	**11.486**$^{\uparrow}$
				0.000	0.003
GJR-t					**29.367**$^{\leftarrow}$
					0.000
Gaussian vs. Student's t models	DCC-t	AsyDCC-t	CCC-t	GARCH-t	GJR-t
DCC	**19.846**$^{\leftarrow}$	**17.636**$^{\leftarrow}$	**10.976**$^{\leftarrow}$	**16.150**$^{\leftarrow}$	**18.345**$^{\leftarrow}$
	0.000	0.000	0.004	0.000	0.000
AsyDCC		**17.233**$^{\leftarrow}$	**6.625**$^{\leftarrow}$	**16.204**$^{\leftarrow}$	**18.031**$^{\leftarrow}$
		0.000	0.036	0.000	0.000
CCC			**8.875**$^{\uparrow}$	**17.995**$^{\leftarrow}$	**16.056**$^{\leftarrow}$
			0.012	0.000	0.000
GARCH				**12.900**$^{\leftarrow}$	**27.291**$^{\uparrow}$
				0.002	0.000
GJR					**29.784**$^{\leftarrow}$
					0.000

Table 2.8: Giacomini and White (2006) CPA test results. Data set: global portfolio. The Table shows test statistics of Giacomini and White (2006) conditional predictive ability test (with p-values). A left (up) arrow means that the model in the row outperforms (underperforms) the model in the column.

Gaussian models	AsyDCC	CCC	GARCH	GJR	CAVIAR
DCC	**4.883**$^{\leftarrow}$	2.728$^{\leftarrow}$	**13.360**$^{\leftarrow}$	**49.510**$^{\leftarrow}$	**11.162**$^{\leftarrow}$
	0.087	0.256	0.001	0.000	0.004
AsyDCC		2.295$^{\leftarrow}$	**13.671**$^{\leftarrow}$	**50.847**$^{\leftarrow}$	**11.038**$^{\leftarrow}$
		0.318	0.001	0.000	0.004
CCC			**13.469**$^{\leftarrow}$	**56.558**$^{\leftarrow}$	**10.374**$^{\leftarrow}$
			0.001	0.000	0.006
GARCH				**69.851**$^{\leftarrow}$	**3.259**$^{\leftarrow}$
				0.000	0.196
GJR					**61.645**$^{\uparrow}$
					0.000

Student's t models	AsyDCC-t	CCC-t	GARCH-t	GJR-t	CAVIAR
DCC-t	**20.533**$^{\leftarrow}$	**20.639**$^{\uparrow}$	**14.317**$^{\leftarrow}$	**69.136**$^{\leftarrow}$	**5.571**$^{\leftarrow}$
	0.000	0.000	0.001	0.000	0.062
AsyDCC-t		**18.935**$^{\uparrow}$	**12.644**$^{\leftarrow}$	**70.958**$^{\leftarrow}$	**4.349**$^{\leftarrow}$
		0.000	0.002	0.000	0.114
CCC-t			**18.352**$^{\leftarrow}$	**64.459**$^{\leftarrow}$	**10.384**$^{\leftarrow}$
			0.000	0.000	0.006
GARCH-t				**89.394**$^{\leftarrow}$	2.885$^{\uparrow}$
				0.000	0.236
GJR-t					**78.280**$^{\uparrow}$
					0.000

Gaussian vs. Student's t models	DCC-t	AsyDCC-t	CCC-t	GARCH-t	GJR-t
DCC	**10.050**$^{\leftarrow}$	**10.223**$^{\leftarrow}$	2.831$^{\leftarrow}$	**15.649**$^{\leftarrow}$	**56.825**$^{\leftarrow}$
	0.007	0.006	0.243	0.000	0.000
AsyDCC		**10.817**$^{\leftarrow}$	2.426$^{\leftarrow}$	**16.229**$^{\leftarrow}$	**58.237**$^{\leftarrow}$
		0.005	0.297	0.000	0.000
CCC			**4.473**$^{\leftarrow}$	**18.360**$^{\leftarrow}$	**64.353**$^{\leftarrow}$
			0.107	0.000	0.000
GARCH				**25.513**$^{\leftarrow}$	**80.937**$^{\leftarrow}$
				0.000	0.000
GJR					**109.325**$^{\leftarrow}$
					0.000

Table 2.9: Giacomini and White (2006) CPA test results. Data set: S&P100 stocks. The Table shows test statistics of Giacomini and White (2006) conditional predictive ability test (with p-values). A left (up) arrow means that the model in the row outperforms (underperforms) the model in the column.

Gaussian models	AsyDCC	CCC	GARCH	GJR	CAViaR
DCC	**14.205**$^{\leftarrow}$	**16.432**$^{\leftarrow}$	**4.727**$^{\leftarrow}$	**13.995**$^{+}$	**10.388**$^{\leftarrow}$
	0.001	0.000	0.094	0.001	0.006
AsyDCC		**16.432**$^{\leftarrow}$	**4.727**$^{\leftarrow}$	**13.995**$^{\leftarrow}$	**10.388**$^{\leftarrow}$
		0.000	0.094	0.001	0.006
CCC			3.894$^{\leftarrow}$	**12.949**$^{\leftarrow}$	**9.800**$^{\leftarrow}$
			0.143	0.002	0.007
GARCH				**16.279**$^{\uparrow}$	**29.379**$^{\leftarrow}$
				0.000	0.000
GJR					**22.241**$^{\leftarrow}$
					0.000

Student's t models	AsyDCC-t	CCC-t	GARCH-t	GJR-t	CAVIAR
DCC-t	**8.166**$^{\uparrow}$	**18.973**$^{\uparrow}$	0.660$^{\leftarrow}$	3.892$^{\leftarrow}$	3.122$^{\leftarrow}$
	0.017	0.000	0.719	0.143	0.210
AsyDCC-t		**18.593**$^{\uparrow}$	0.664$^{\leftarrow}$	**5.714**$^{\leftarrow}$	3.420$^{\leftarrow}$
		0.000	0.718	0.057	0.181
CCC-t			**7.185**$^{\leftarrow}$	**15.883**$^{\leftarrow}$	**9.993**$^{\leftarrow}$
			0.028	0.000	0.007
GARCH-t				**24.386**$^{\uparrow}$	**31.471**$^{\leftarrow}$
				0.000	0.000
GJR-t					**30.572**$^{\leftarrow}$
					0.000

Gaussian vs. Student's t models	DCC-t	AsyDCC-t	CCC-t	GARCH-t	GJR-t
DCC	**18.752**$^{\leftarrow}$	**16.908**$^{\leftarrow}$	**2.730**$^{\leftarrow}$	**7.429**$^{\leftarrow}$	**16.429**$^{\leftarrow}$
	0.000	0.000	0.255	0.024	0.000
AsyDCC		**16.908**$^{\leftarrow}$	2.730$^{\leftarrow}$	**7.429**$^{\leftarrow}$	**16.429**$^{\leftarrow}$
		0.000	0.255	0.024	0.000
CCC			**8.386**$^{\uparrow}$	**6.670**$^{\leftarrow}$	**15.652**$^{\leftarrow}$
			0.015	0.036	0.000
GARCH				**24.785**$^{\leftarrow}$	**12.802**$^{\leftarrow}$
				0.000	0.002
GJR					**28.130**$^{\leftarrow}$
					0.000

Table 2.10: Basel II penalty zones

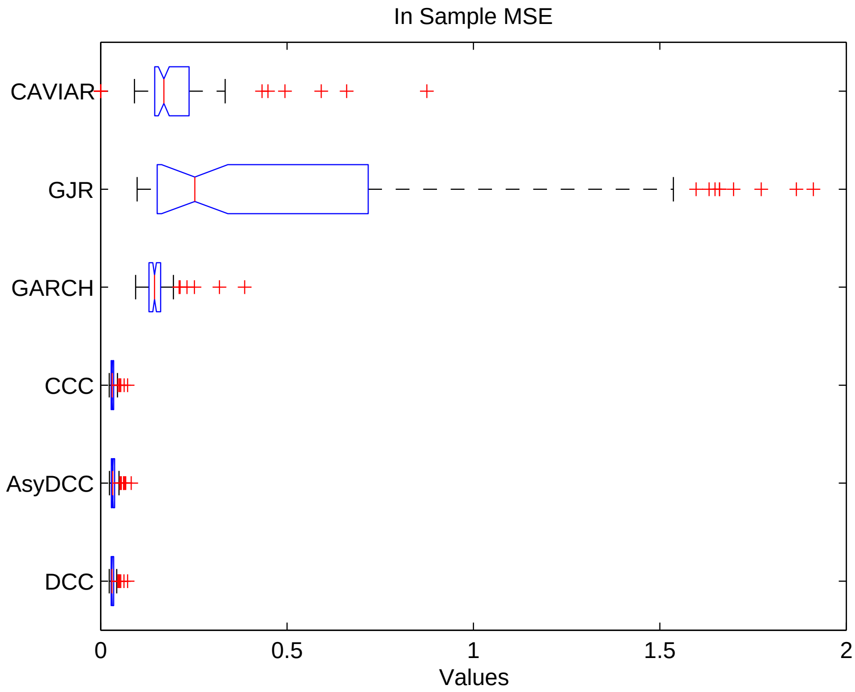
Zone	Number of violations	Increase in k
Green	0-4	0.00
Yellow	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85
Red	>10	1.00

Note: based on the number of violations for 250 business days.

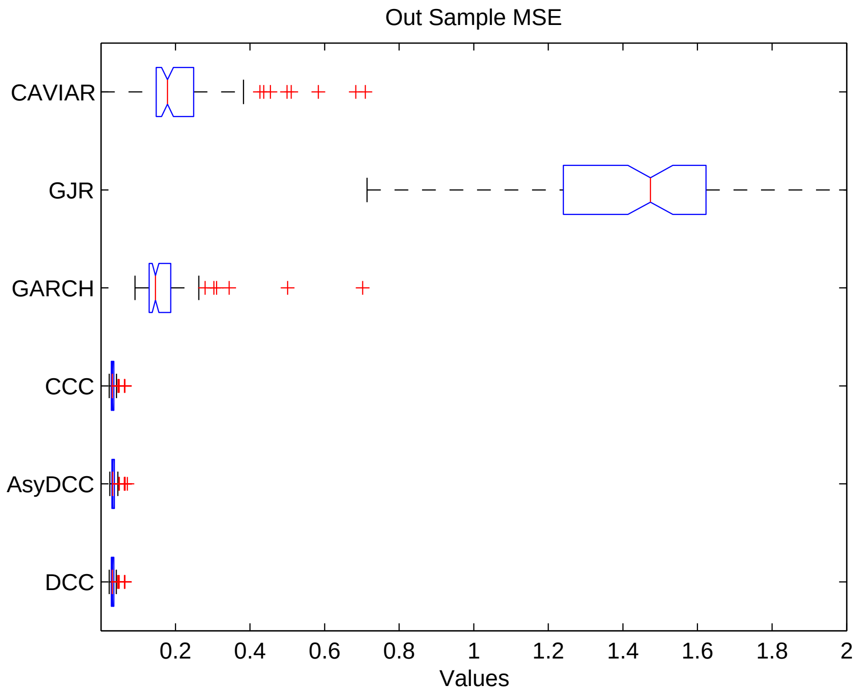
Table 2.11: Mean daily capital requirements and number of VaR violations

	Mean daily capital requirement (%)			VaR violations first 250 out-sample obs.			VaR violations last 250 out-sample obs.		
	Industry portfolios	Global portfolio	S&P100 stocks	Industry portfolio	Global portfolio	S&P100 stocks	Industry portfolios	Global portfolio	S&P100 stocks
Gaussian models									
DCC	5.02	3.06	4.61	7	5	5	15	10	14
AsyDCC	5.04	3.08	4.61	7	5	5	15	10	14
CCC	5.10	3.04	4.66	7	4	5	13	9	13
GARCH	5.30	3.31	5.10	5	3	5	11	8	8
GJR	5.29	3.67	5.17	1	1	3	9	2	9
CAVIAR	5.24	3.34	5.13	5	4	4	11	5	5
Student's t models									
DCC-t	4.95	3.07	5.17	4	3	5	12	7	12
AsyDCC-t	4.95	3.10	5.11	4	3	5	12	6	12
CCC-t	5.05	3.05	4.58	7	5	5	13	8	14
GARCH-t	5.34	3.44	5.15	5	3	4	10	7	7
GJR-t	5.30	3.82	5.20	1	1	2	9	2	8
CAVIAR	5.24	3.34	5.13	5	4	4	11	5	5

Figure 2.1: Monte Carlo distribution of the mean squared error (MSE) of estimated VaR

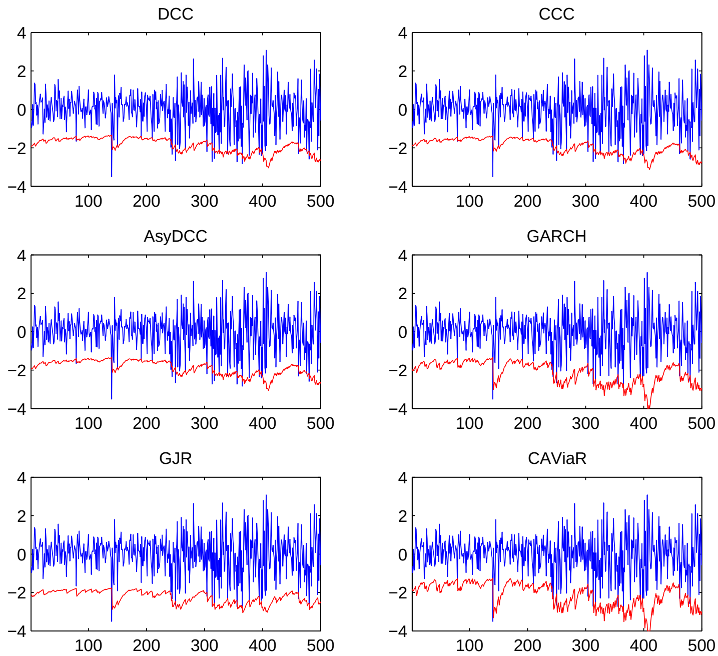


(a) In-sample

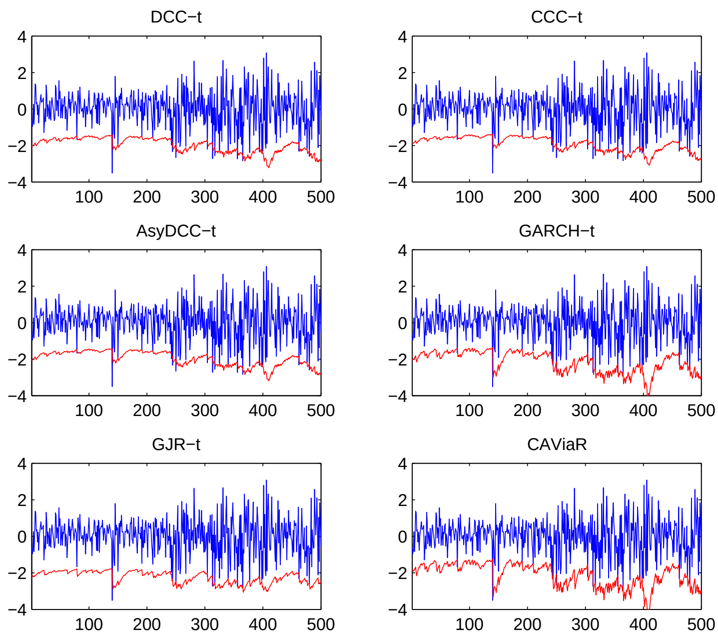


(b) Out-of-sample

Figure 2.2: Out-of-sample estimated VaRs for the industry portfolios

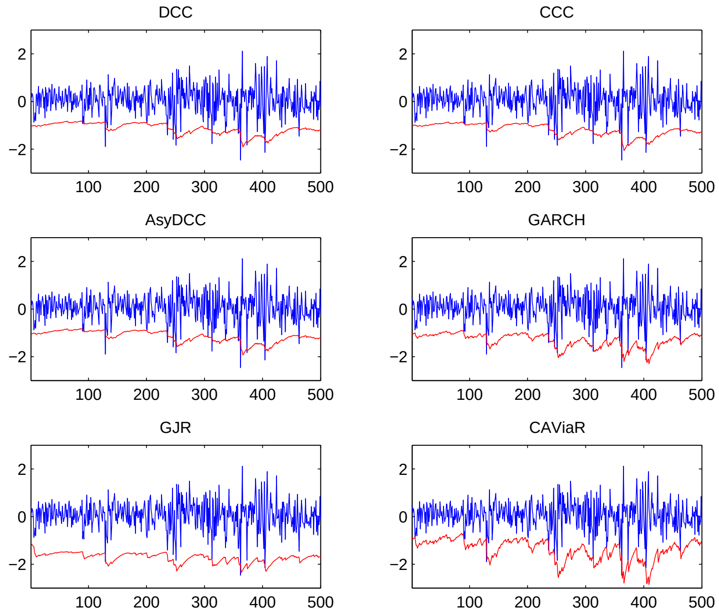


(a) Normally distributed models

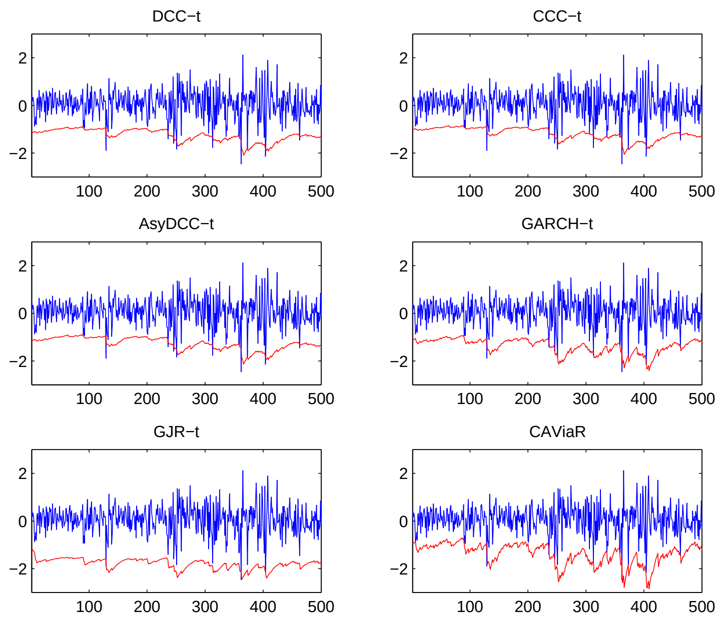


(b) Student's *t* distributed models

Figure 2.3: Out-of-sample estimated VaRs for the global portfolio

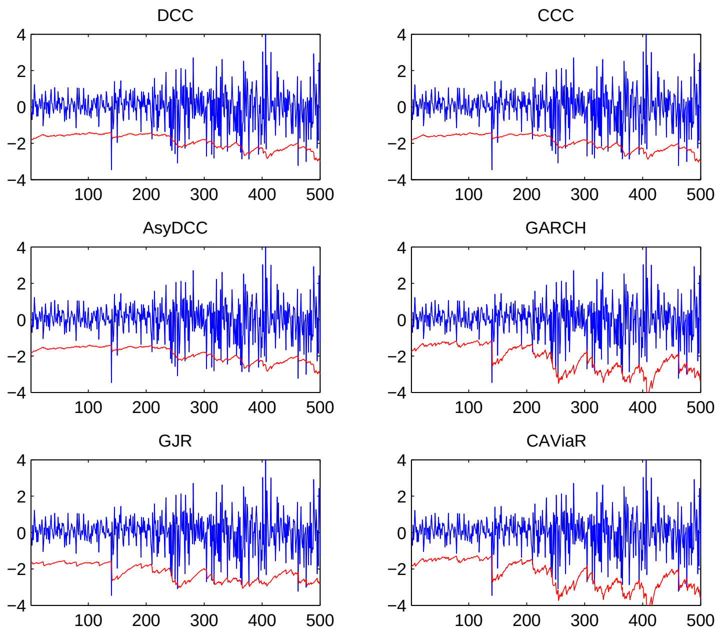


(a) Normally distributed models

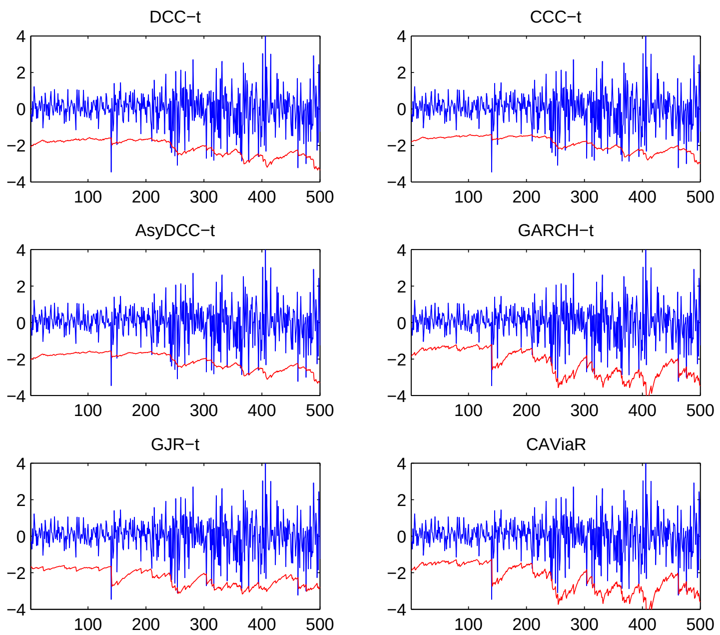


(b) Student's *t* distributed models

Figure 2.4: Out-of-sample estimated VaRs for the S&P 100 stocks



(a) Normally distributed models



(b) Student's t distributed models

Chapter 3

Optimal portfolios with minimum capital requirements

3.1 Introduction

The Basel II framework (Bank for International Settlements 2006) requires banks to set aside a minimum amount of regulatory capital to cover potential losses arising from their exposure to credit risk, operational risk and market risk. Market risk is the risk of losses on positions in equities, interest rate related instruments, currencies and commodities due to adverse movements in market prices. The capital requirements for market risk are based upon estimates of the Value-at-Risk (VaR), defined as the maximum loss on the bank's positions in these assets that could occur over a given holding period with a specified confidence level.

Basel II allows banks to use 'internal' models to measure their VaR, as an alternative to the standardized approach described in the accord (Hendricks and Hirtle 1997). This standardized approach is known to render conservative VaR measures, leading to excessively high capital requirements. From the banks' perspective this is undesirable given that, among others, regulatory capital involves an opportunity cost as it cannot be used for other, profitable purposes. Hence, it is attractive for banks to attempt to lower their capital charges through their own risk management system. The empirical evidence presented by Pérignon et al. (2008) suggests that the use of internal models indeed is widespread among large financial institutions.

Although internal risk measurement systems are subject to supervisory approval based on qualitative and quantitative standards, banks enjoy a large degree of freedom in devising the precise nature of their models. This flexibility does not, however, imply that banks are tempted to pursue the lowest possible VaR estimates. This is due to the fact that the relation between VaR estimates and capital requirements is non-monotonic, as it takes into account not only the magnitude of the VaR measures but also the number of violations of the VaR (i.e. actual losses exceeding the VaR) in the recent past. Specifically, the regulatory capital requirement to be held on day $t+1$ is determined as the maximum of the current VaR estimate and the average VaR over the preceding 60 business days multiplied by a scaling factor, that is,

$$\text{Capital Requirement}_{t+1} = \max\left\{\text{VaR\$}_t, (3+k) \times \overline{\text{VaR\$}_{t,60}}\right\} \tag{3.1}$$

where $\text{VaR\$}_t$ is the estimate at day t of the VaR for a and $\overline{\text{VaR\$}_{t,60}} = \frac{1}{60}\sum_{j=0}^{59} \text{VaR\$}_{t-j}$.[1] The penalty or “plus” k in the multiplication factor in (3.1) ranges between 0 and 1. Its exact value is determined by the number of VaR violations during the last 250 business days, as shown in Table 2.10. As we noted before, The Basel accord requires the use of VaR estimates for a holding period h of 10 days at confidence level α of 1%. The accord, however, allows the 10-day VaR estimates to be computed from VaR estimates for shorter periods by using the square-root-of-time-rule.

The expression for the capital requirements in (3.1) seemingly suggests that lower capital charges could be achieved by lower VaR estimates. This, however,

[1]Note that these VaR estimates are expressed in dollar terms, representing the loss that might be incurred on the current portfolio; that is, $\text{VaR\$}_t = V_t(1 - e^{\text{VaR}_t})$ with V_t being the current portfolio value and VaR_t the VaR in terms of returns. Usually it is the latter VaR that is first obtained from a model for the portfolio return distribution, and we follow this practice here. Moreover, we also follow the practice of expressing the VaR in terms of returns as a negative number. This is the case when the interest is in estimating the risks associated to decreases in asset prices.

need not be the case as lower VaR estimates are possibly violated more often, thus increasing the regulatory capital through the effects of the penalty factor k, see Table 2.10. Apart from direct costs due to the larger amount of capital that needs to be put aside, this may also bring indirect costs by damaging the bank's reputation. Both types of costs become particularly severe when the 'red zone' is entered, that is, when ten or more VaR violations occur during a period of 250 business days. In that case, the bank may be forced to adopt the Basel accord's standardized approach for VaR estimation. As noted before, this approach is known to render conservative VaR measures, leading to excessively high capital requirements. In addition, the ban of the bank's internal models obviously has detrimental effects on its reputation.

In practice banks appear to be wary of being overly optimistic about their level of market risk. In fact, empirical evidence presented by Berkowitz and O'Brien (2002), Pérignon et al. (2008) and Pérignon and Smith (2009) suggests that they systematically *overestimate* their VaR. For instance, Berkowitz and O'Brien (2002) document that the number of violations of VaR estimates at the 1% level of six large US banks is usually lower than expected. Similarly, for VaR estimates of the six largest Canadian banks Pérignon et al. (2008) report that during the 7,354 trading days analyzed, there are only two violations whereas the expected number is 74.

The exaggeration of banks' own level of risk implies an excessive amount of regulatory capital, directly affecting the profitability of the bank. Another, at least as undesirable consequence is that such banks appear more risky than they actually are, thus generating reputational concerns about their risk management systems. This affects investors' perception and can induce underinvestment in VaR-overstating banks. Indeed, Jorion (2002) found that the VaR disclosures are informative about the future variability in trading revenues, thus corroborating the idea that analysts/investors may be using the VaR disclosures to support investment decisions.

In this chapter we put forward a novel portfolio construction methodology to overcome the drawbacks of both over- and understatement of a bank's VaR as discussed above. Specifically, we propose to determine optimal portfolio weights by directly minimizing the daily capital requirements, but subject to a restriction on the number of VaR violations during the preceding year. Implicitly, our approach aims to find the optimal balance between the level of VaR measures and the number of VaR violations, thus leading to the lowest possible level of capital requirements.

Our proposed methodology differs in important ways from previous, related research. First, in order to achieve the goal of lower capital requirements, one possibility is to develop a VaR model that delivers lower levels of capital charges, as proposed recently by McAleer et al. (2010), for instance. Using the terminology of Christoffersen (2009), this approach can be considered a risk measurement or *passive* risk management problem, since it is applied to a given (i.e. predetermined) portfolio composition. Alternatively, in this paper we propose to perform *active* risk management by deciding on the portfolio allocations themselves to attain lower levels of capital requirements. Second, portfolios with low levels of capital requirements may be obtained by imposing constraints or targets on the amount of capital requirements or on the portfolio VaR (for instance, a VaR cap), as in Sentana (2003), Cuoco and Liu (2006) and Alexander et al. (2007). In our approach, the level of capital requirements plays a much more central role as it is taken to be the objective function that should be minimized.

Although minimizing the capital requirements might be an important criterion to take into account, in real world situations portfolio managers and investors traditionally consider standard performance measures, such as returns or Sharpe ratios, to decide on optimal allocations. Therefore, we consider a more general formulation of the portfolio construction problem in which the optimal portfolio composition is found by minimizing the level of capital requirements subject also to a given (i.e. user specified) target performance, measured in terms of returns.

We apply the proposed methodology to three asset portfolios detailed in section 2.5. The minimum capital requirement portfolio is compared to various benchmark portfolios, including the minimum-VaR portfolio and the equally weighted portfolio. The results for the futures portfolio indicate that our approach delivers lower capital requirement levels in comparison to the benchmark portfolios, along with a lower average number of VaR violations. For the portfolios of sector indices and S&P 100 stocks, the novel portfolio construction approach delivers a better balance between capital requirement levels and the number of VaR violations. For all three data sets, we find a lower average number of VaR violations for the MCR portfolios. In addition for the MCR portfolios the number of VaR violations does not enter the red zone in the vast majority of the specifications. This is in sharp contrast to the benchmark portfolios, for which we frequently find more than ten VaR violations. Finally, the performance of the MCR portfolios in terms of gross returns, risk-adjusted returns, portfolio turnover, and break even transaction costs are superior to those of minimum-VaR portfolios.

The remainder of the chapter is organized as follows. Section 3.2 describes the procedure to obtain optimal portfolios with minimum capital requirements. Section 3.3 discusses the methodology for the out-of-sample evaluation and the benchmark portfolios, and presents the empirical applications. Finally, Section 3.4 concludes.

3.2 Optimal Portfolios with Minimum Capital Requirements

The main ingredient required to obtain optimal portfolios with minimum capital requirements (hereafter MCR portfolios) is a measure of the VaR. We obtain VaR estimates using the methodology previously discussed in subsection 1.1. It is worth noting, however, that the MCR portfolio construction methodology developed in the remainder of this section is independent of the method used to obtain the VaR measures. Nevertheless, and perhaps obviously, we may expect that more accurate modeling of the expected returns and conditional covariance matrix leads to improved portfolio characteristics. Finally, in section 3.2.1 we then develop the optimization problem that leads to the construction of MCR portfolios.

3.2.1 MCR Portfolios

The problem of constructing an optimal MCR portfolio consists of finding the vector of portfolio weights w that minimizes the capital charges in (3.1) subject to a restriction on the number of VaR exceptions during the previous 250 trading days and other constraints. We now describe the objective function as well as the restrictions involved in the optimization problem for MCR portfolios in more detail.

Objective function

The objective function for constructing an optimal MCR portfolio consists of minimizing the amount of regulatory capital in (3.1). Here we make the simplifying assumption that instead of the VaR expressed in terms of portfolio losses, we focus on the VaR expressed in terms of portfolio returns. Note that this implies taking the minimum of the current VaR estimate and the average over the previous 60 business days. Using the VaR definition in (1.5) and the expressions for the conditional mean and variance of the portfolio returns in (1.3) and (1.4), we can then write the objective function as

$$\underset{w}{\text{minimize}} \max \left\{ -(w'\mu_{t+1} + (w'H_{t+1}w)^{1/2}q), -\frac{(3+k)}{60} \sum_{j=0}^{59} (w'\mu_{t+1-j} + (w'H_{t+1-j}w)^{1/2}q) \right\}, \tag{3.2}$$

where the decision variable is the $N \times 1$ vector of portfolio weights w at time t. As we mentioned before, we express the VaR in terms of returns a negative number. Therefore, we put a minus sign in front of each term in (3.2) to obtain positive numbers and to be consistent with the capital requirement formula in (3.1). Note that for computing the average VaR over the previous 60 business days we need the historic one-step-ahead forecasts for the conditional mean μ_{t+1-j} and for the conditional covariances H_{t+1-j}, for $j = 0, \ldots, 59$. Even more important to note is that the average VaR is hypothetical, in the sense that it is based on the portfolio with the weights that are currently determined for day $t+1$.

Restriction on the number of VaR violations

A VaR violation occurs when the portfolio return on a given trading day falls below the VaR estimate. The occurrence of a VaR violation on day $t+1$ can be characterized by means of an indicator function, $\mathbf{1}\big(w'R_{t+1} < w'\mu_{t+1} + (w'H_{t+1}w)^{1/2}q\big)$, which takes the value 1 when the argument is true, i.e. when a VaR violation occurs. We are interested in restricting the number of VaR violations over the last 250 trading days to be less than or equal to a certain threshold δ. Therefore, we can write this restriction as:

$$\sum_{j=1}^{250} \mathbf{1}\Big(w'R_{t+1-j} < w'\mu_{t+1-j} + (w'H_{t+1-j}w)^{1/2}q\Big) \leq \delta. \tag{3.3}$$

The value of δ can be chosen by taking into consideration the penalties reported in Table 2.10. For instance, if the interest is to avoid the number of VaR violations reaching the "red zone", then we should set $\delta = 9$. Similar to the average VaR over the previous 60 business days, the number of VaR violations over the previous 250 days in (3.3) is hypothetical, in the sense that it is based on the portfolio with the weights that are currently determined for day $t+1$.[2]

Target performance

In many practical situations involving portfolio selection, investors and/or portfolio managers are interested in achieving a certain target performance. For that

[2]Of course, the ex-post evaluation of the portfolio in terms of the number of VaR violations and capital requirements is based on actual portfolios, therefore in accordance to the criteria established by Basel II. Further implementation details are discussed in Section 3.3.3.

purpose, we incorporate the following restriction on the expected portfolio returns:

$$w'\mu_{t+1} \geq \Xi, \tag{3.4}$$

where Ξ denotes the desired target performance. Note that alternative specifications for restrictions on the target performance can be considered, such as a constraint on the Sharpe ratio, on the portfolio turnover, or on the tracking error; see Cornuejols and Tütüncü (2007).

Constraints on the portfolio weights

Finally, restrictions often are imposed on the portfolio weights, for example to avoid short-selling or to achieve a minimum diversification level. Previous research has shown that imposing such constraints may substantially improve performance, mostly by reducing risk, see Jagannathan and Ma (2003), among others. For this reason, we allow for a general set of constraints on the portfolio weights:

$$w \in \Omega, \tag{3.5}$$

where Ω represents the set of allowable portfolio weights as defined by the specific restrictions that are imposed. For instance, if short-selling should be avoided, we may specify the restriction $w \geq 0$. Similarly, a certain level of diversification may be guaranteed by imposing an upper bound on the individual portfolio weights, i.e. $w_i \leq u$ for some $0 < u < 1$.[3] Finally, we may achieve full investment by restricting the portfolio weights to sum up to 1, i.e. $\iota' w = 1$, where ι is a vector of ones.

The optimization problem as formulated in (3.2)-(3.5) is very general as it allows i) different econometric specifications for the expected returns and for the conditional covariance matrix, ii) different threshold levels for the maximum number of VaR violations, iii) different types of restrictions on the target performance, and

[3] DeMiguel et al. (2009) recently proposed a unifying approach based on constraints of the portfolio norms that nests several commonly applied restrictions as special cases, including the no-shortselling and diversification constraints.

iv) various different constraints on the portfolio weights. Albeit very general, the formulation in (3.2)-(3.5) has a major shortcoming: the objective function and the restriction on the number of VaR violations are both discontinuous and non-convex due to the presence of the operator `max` and the indicator function, respectively. Note that the non-convexity imposes important difficulties in terms of computational effort and a potential problem of local minima; see Nocedal and Wright (1999) and Boyd and Vandenbergue (2004). For this reason we next formulate a convex and continuous approximation to the If original problem for which a highly accurate solution can be obtained with low computational effort.

3.2.2 A convex and continuous reformulation

Reformulating the objective function

In order to obtain a continuous and smooth objective function, (3.2) can be reformulated by introducing an artificial variable to get rid of the operator `max`; see Nocedal and Wright (1999, chapter 12). Specifically, the objective function in (3.2) can be equivalently expressed as the following linear optimization problem:

$$\begin{aligned} &\underset{w,v}{\text{minimize}}\, v \qquad (3.6)\\ &\text{subject to:}\\ &v \geq -(w'\mu_{t+1} + (w'H_{t+1}w)^{1/2}q)\\ &v \geq -\frac{(3+k)}{60}\sum_{j=0}^{59}(w'\mu_{t+1-j} + (w'H_{t+1-j}w)^{1/2}q), \end{aligned}$$

thus yielding a continuous and convex expression. Note that the attractiveness of the equivalent reformulation in (3.6) is that it replaces the minimization of a nonlinear, non-smooth objective function by the minimization of a smooth, linear objective function with convex constraints (Nocedal and Wright 1999).

Reformulating the restriction on the number of VaR violations

Due to the presence of an indicator function, the original restriction on the number of VaR violations in (3.3) is non-differentiable, discontinuous and non-convex. Boyd and Vandenbergue (2004) propose a *convex approximation* by eliminating the indicator function while keeping its argument, which leads to

$$\frac{1}{250}\sum_{j=1}^{250}\left(w'\mu_{t+1-j} + (w'H_{t+1-j}w)^{1/2}q - w'R_{t+1-j}\right) < \tilde{\delta}, \tag{3.7}$$

where $\tilde{\delta}$ is a parameter that must be calibrated in order to achieve the desired results regarding the number of VaR violations over the last 250 observations. The procedure to calibrate this parameter is detailed in Subsection 3.3.3. Note that under this approximation, our displeasure regarding a VaR violation grows as the constraint becomes "more violated". In other words, this approximation implies that VaR violations of greater magnitude are more penalized than those of less magnitude, which makes sense from a practical point of view. Note that this feature is not captured by the indicator function, since the VaR violations of smaller magnitude will have the same importance as those of greater magnitude. Moreover, the proposed approximation is the best convex approximation to (3.3); see Boyd and Vandenbergue (2004).

In sum, based on the reformulation of the objective function in (3.6) and of the constraint on the number of VaR violations in (3.7), the original optimization problem in (3.2)-(3.5) admits the following convex reformulation:

$$
\begin{aligned}
&\underset{w,v}{\text{minimize}}\, v \qquad\qquad (3.8)\\
&\text{subject to:}\\
&v \geq -(w'\mu_{t+1} + (w'H_{t+1}w)^{1/2}q)\\
&v \geq -\frac{(3+k)}{60}\sum_{j=0}^{59}(w'\mu_{t+1-j} + (w'H_{t+1-j}w)^{1/2}q)\\
&\frac{1}{250}\sum_{j=1}^{250}\left(w'\mu_{t+1-j} + (w'H_{t+1-j}w)^{1/2}q - w'R_{t+1-j}\right) < \tilde{\delta}\\
&w'\mu_{t+1} \geq \Xi\\
&w \in \Omega.
\end{aligned}
$$

Note that the optimization problem in (3.8) is a *second order cone formulation* (Nocedal and Wright 1999; Boyd and Vandenbergue 2004; Grant and Boyd 2008). Therefore, the problem can be accurately solved in practice with low computational effort.

Finally, one last technical comment concerning the optimization problem in (3.8) is that, since the MCR portfolios are dynamic in the sense that the optimal portfolio weights have to be re-calculated on a daily basis, the penalty parameter k has also to be updated based on the number of VaR violations over the last 250 observations. This means that a minimum of 250 realizations of the optimal MCR portfolio returns and VaRs are required to start evaluating and updating k. Therefore, in order to be conservative, over the first 250 observations we set k=1 in the computation of the capital requirements, which is the highest value that this penalty parameter can assume. After the 250th realization, we update k according to the values presented in Table 2.10. Moreover, to ensure a consistent portfolio evaluation we focus on the capital requirements and the number of VaR violations obtained after the 250th realization, since a minimum of 250 realizations is required to evaluate the ex-post performance of actual portfolios in accordance to the criteria established by Basel

II.

3.3 Empirical Application

We evaluate the performance of the MCR strategy for the three portfolios with different types of assets considered in the previous chapter. In the optimization problem in (3.8), we adopt a daily target portfolio return of $\Xi = 4$ bp, corresponding to an annual target return of 10%. Furthermore, we impose a no-shortselling restriction, $w \geq 0$, thus focusing on the case in which only long positions are allowed.

3.3.1 Expected returns and conditional covariances

Computing a VaR measure for a given portfolio requires estimates of the expected returns and the conditional covariance matrix of the included assets, see (1.5). In our empirical application these inputs are obtained from multivariate parametric models.

Expected returns are obtained from a VAR(1) model for the return vector R_{t+1},

$$R_{t+1} = A + BR_t + \varepsilon_{t+1} \tag{3.9}$$

where A is an $N \times 1$ vector of constants, B is the $N \times N$ autoregressive matrix, and ε_t is a vector of shocks (or unexpected returns), which are assumed to be uncorrelated and identically distributed as multivariate Normal with a positive definite covariance matrix Σ_ε. We assume that R_t is a covariance-stationary process with mean $\mu = E(R_t)$ and positive definite autocovariance matrix $\Gamma_l = E[(R_{t-l} - \mu)(R_t - \mu)']$, for $l = 0, 1$. In the subsection 3.3.6 we report the results of robustness checks considering a particular case of (3.9) in which the matrices A and B are assumed to be diagonal. In this case we have an univariate AR(1) model for each of the individual assets in the system.

In order to obtain estimates of the second order moments, we consider the multivariate econometric specification discussed in section 1.1.3. In particular, we consider the Risk Metrics, DCC, and CCC models. Moreover, we also consider the (unconditional) shrinkage estimator of the sample covariance matrix proposed by Ledoit and Wolf (2003) due to its ability of dealing with the estimation error in large covariance matrices.

3.3.2 Benchmark portfolios

We consider two alternative benchmarks for the purpose of comparison with the proposed MCR portfolios. The first benchmark is a minimum-VaR (Min-VaR) portfolio. This is obtained from an optimization problem in which the investor wishes to perform active portfolio management by minimizing the portfolio VaR subject to a target return and possible other restrictions.[4] The Min-VaR optimization problem is given by

$$
\begin{aligned}
&\underset{w}{\text{minimize}} \; -\left(w'\mu_{t+1} + (w'H_{t+1}w)^{1/2}q\right) \qquad (3.10)\\
&\text{subject to:}\\
&\quad w'\mu_{t+1} \geq \Xi\\
&\quad w \in \Omega.
\end{aligned}
$$

In (3.10), we adopt the same target return and the no-shortselling restriction used to obtain MCR portfolios.

As a second benchmark we consider the equally weighted (or $1/N$) portfolio, which has been extensively studied in the empirical literature. For instance, DeMiguel et al. (2009) found that the $1/N$ portfolio outperformed (in terms or

[4]The properties of the Min-VaR portfolio have been extensively studied by Alexander and Baptista (2002). It is shown that the solution to the VaR minimization problem is always distinct from the solution to the variance minimization problem. Moreover, the Min-VaR portfolio at the 99% confidence level is a mean-variance efficient portfolio with expected returns greater than the expected return of the min-variance portfolio.

Sharpe ratio and turnover) 14 widely used portfolio strategies, such as mean-variance and minimum variance. Therefore, it seems natural to compare our results against this simple, but powerful portfolio in which all the assets in the portfolio have the same weight. We note that for the $1/N$ portfolio, portfolio returns are independent of the method used to model and forecast the expected returns and the conditional covariance matrix. On the other hand, the VaR esimtate, the level of capital requirements, and the number of VaR violations are affected by these methods.

3.3.3 Implementation details

We use a rolling estimation window of $\tau = 1000$ observations to estimate the parameters of the models that are used for generating the expected returns and the conditional covariance matrix. The following stepwise procedure then can be used to obtain optimal MCR portfolios:

1. Using the observations for $t = 1, \ldots, \tau$, estimate the coefficients in the VAR(1) model (3.9) and in the model for the conditional covariance matrix.

2. Compute the expected return $\mu_{\tau+1}$ and the conditional covariance matrix $H_{\tau+1}$.

3. Use the last 250 observations up to observation τ to solve the optimization problem in (3.8) and obtain the optimal MCR portfolio weights, w_τ;

4. Compute the portfolio return for day $\tau + 1$ as $r_{p,\tau+1} = w'_\tau R_{\tau+1}$, and the portfolio VaR as $VaR_{\tau+1} = w'_\tau \mu_{\tau+1} + (w'_\tau H_{\tau+1} w_\tau)^{1/2} q$;

5. Move to the next window with observations $t = 2, \ldots, \tau + 1$ and repeat steps 1 to 3 until the end of the sample is reached.

After completing these steps, we have a total of $T - \tau$ out-of-sample observations for the portfolio return and one-step-ahead estimates of the portfolio VaR, where T denotes the sample size.

It is useful to note the following two points, related to our choice of models for the conditional covariance matrix. First, the Risk Metrics approach does not involve unknown coefficients as we set $\lambda = 0.94$. Second, the shrinkage estimator of Ledoit and Wolf (2003) assumes that the covariance matrix is constant during the estimation window. When the shrinkage estimator is computed using an estimation window up to observation τ, we set the conditional covariance matrix $H_{\tau+1}$ equal to the resulting estimate.

The performance of the proposed MCR portfolios depends on a good choice of the parameter $\tilde{\delta}$ in (3.7), which controls the desired maximum number of VaR violations. In order to calibrate this parameter, we use a cross validation procedure - see Efron and Gong (1983) for a detailed explanation and DeMiguel et al. (2009) for an application in the context of portfolio optimization. In order to perform the cross validation we first select for each data set the first $L = 1000$ observations as the *training set*. Second, we fit a multivariate model for the conditional mean and for the conditional covariance using the $L = 1000$ observations in the training set. Third, starting from the first observation, we perform the iterative algorithm described above using an estimation window of $\tau = 250$ observations until the end of the training set is reached. To alleviate the burden of the calibration process, in the iterative algorithm we do not re-estimate the parameters of the multivariate models. Finally, based on the $L - \tau$ observations obtained in the training set, we pick the value of $\tilde{\delta}$ that minimizes the average capital requirements such that the maximum number of VaR violations is less or equal than 9, which is the upper bound for the "yellow zone" according to Table 2.10. The value selected for the parameter $\tilde{\delta}$ is used to obtain the MCR portfolios for the remaining observations of the data set. This process is repeated for each of the multivariate models for the covariance matrices discussed in section 1.1.

3.3.4 Out-of-sample evaluation

Most important for the evaluation of the MCR portfolios are the characteristics of the daily capital requirement (DCR) and the number of VaR violations, both in an absolute sense and compared to the benchmark portfolios. For each specification of the covariance matrix (DCC, RM and LW) we consider the mean daily capital requirement ("mean DCR"), the average number of VaR violations ("Mean Hit"), the maximum number of VaR violations ("Max Hit"), and the fraction of days for which the number of VaR violations is either within the "green zone" (i.e. below 5) or within "red zone" (i.e. above 9). The statistics concerning the VaR exceedances are based on rolling periods of 250 out-of-sample observations, which is the time period established by the Basel II to evaluate the financial institutions' VaR disclosures.

We also examine the portfolios' performance in terms of gross returns, standard deviation of returns, turnover, and Sharpe ratio. Specifically, we compute the following statistics (based on the out-of-sample observations): mean portfolio return, standard deviation of portfolio returns, Sharpe ratio (SR) and portfolio turnover. The statistics are defined as:

$$\begin{aligned} \hat{\mu} &= \frac{1}{T-\tau}\sum_{t=\tau}^{T-1} w_t' R_{t+1} \qquad (3.11) \\ \hat{\sigma} &= \sqrt{\frac{1}{T-\tau-1}\sum_{t=\tau}^{T-1} (w_t' R_{t+1} - \hat{\mu})^2} \\ \widehat{SR} &= \frac{\hat{\mu}}{\hat{\sigma}} \\ \text{Turnover} &= \frac{1}{T-\tau-1}\sum_{t=\tau}^{T-1}\sum_{j=1}^{N}\left(\left| w_{j,t+1} - w_{j,t^+},\right|\right) \end{aligned}$$

where w_{j,t^+} is the portfolio weight in asset j at time $t+1$ but *before* rebalancing and $w_{j,t+1}$ is the desired portfolio weight in asset j at time $t+1$. As pointed out by DeMiguel et al. (2009), the turnover can be interpreted as the average percentage of wealth traded in each period.[5]

To measure the impact of transaction costs on the performance of the different portfolios (Han 2006; DeMiguel et al. 2010), we consider the out-of-sample portfolio mean returns net of transaction costs, μ_{TC}, defined as:

$$\hat{\mu}_{TC} = \frac{1}{T-\tau} \sum_{t=\tau}^{T-1} \left[\left(1 + w_t' R_{t+1}\right) \left(1 - c \sum_{j=1}^{N} \left| w_{j,t+1} - w_{j,t+} \right| \right) - 1 \right] \tag{3.12}$$

where c is the fee to be paid for each transaction. Instead of assuming an arbitrary value of c, we report the value of the *break even* transaction cost. In other words, we report the value of c such that the portfolio mean return net of transaction costs is equal to zero. Therefore, when comparing two alternative portfolio strategies, the one with a higher break even cost is an outperformer as it is necessary a higher transaction cost to break even the portfolio net returns.

To test the hypothesis that the capital requirement levels, the number of VaR exceedances, and the Sharpe ratios obtained with the MCR portfolios and with the benchmark portfolios are equal, we follow DeMiguel et al. (2009) and use the stationary bootstrap of Politis and Romano (1994) with B=1,000 bootstrap resamples and expected block size b=5.[6] The resulting bootstrap p-values are obtained using the methodology suggested in Ledoit and Wolf (2008, Remark 3.2).

3.3.5 Results

Table 3.1 reports the average daily capital requirements and the number of VaR violations obtained for the MCR, the Min-VaR and the $1/N$ portfolio strategies. The first striking result for the 30-asset Global portfolio as reported in Panel A is

[5]Note that, in the case of an equally weighted (or $1/N$) portfolio composition, we have $w_{j,t} = w_{j,t+1} = 1/N$, but w_{j,t^+} may be different due to changes in asset prices between t and $t+1$.

[6]We performed extensive robustness checks regarding the choice of the block length used to test the differences in capital requirements, VaR exceedances and Sharpe ratios. In particular, we compared the p-values obtained by using block lengths ranging from $b = 5$ to $b = 250$. The results are robust to the choice of the block length and similar to those reported here.

that for all specifications of the conditional covariance matrix the MCR portfolio delivers significantly lower capital requirements in comparison to the benchmark portfolios. Moreover, the average number of VaR exceedances is significantly lower in two out of three cases, leading to a fraction of days within the “green zone” above 90% in all specifications for the MCR portfolios. The best result in terms of capital requirements is achieved when the RM estimator is used (1.099%) whereas the best result in terms of average number of VaR violations is obtained when the LW shrinkage estimator is used (1.180). The maximum number of VaR violations does not enter into the “red zone” for the MCR and Min-VaR portfolios. This does not apply to the $1/N$ portfolio in particular when the shrinkage estimator is used, for which the number of violations entered in the “red zone” in approximately 50% of the days. In sum, the results in Panel A indicate that a financial institution applying the MCR portfolio methodology for this set of assets would have enjoyed lower capital charges along with a very reasonable number of VaR exceedances.

The results for the 48 industry portfolios in panel B of Table 3.1 are very useful to illustrate the trade-off between capital requirement levels and the number of VaR violations. At first glance, one could argue that the MCR portfolios do not perform well, in the sense that they render higher DCR than both benchmark portfolios. This conclusion is, however, misleading: The lower capital charges for the benchmark portfolios come at the expense of a very high number of VaR violations, leading to a high fraction of days within the “red zone”. As discussed before, this is highly undesirable due to potential damaging effects on the banks’ reputation regarding their risk management systems. For instance, using the DCC model for the Min-VaR portfolio delivers an average and maximum number of VaR violations equal to 9.13 and 17, respectively, leading to a total of 28% of days in the “red zone”. On the other hand, for the MCR strategy the corresponding numbers are 3.59 and 6, altogether avoiding the “red zone” during the whole out-of-sample period.

The results for the S&P100 stocks reported in Panel C of Table 3.1 are, in general, similar to those obtained with the industry portfolios. Again the MCR portfolios achieve a better balance between capital requirement levels and the number of VaR exceedances. In all cases the average number of VaR violations for the MCR portfolios is significantly lower than that of the Min-VaR portfolio. All benchmark specifications deliver an average number of VaR violations that is well into the "yellow zone" (or even in the "red zone" for the Min-VaR portfolio when the RM estimator is used). Therefore, the fraction of days for these portfolios entered into the "red zone" is high (approximately 70% for the Min-VaR portfolios using the RM estimator). In stark contrast, for the MCR portfolios the average number of VaR violations is close to the lower bound of the "yellow" zone such that a non-zero penalty factor occurs only on a limited fraction of days. Only when the LW shrinkage estimator is used does the maximum number of VaR violations exceeds the "yellow zone" upper bound (but at a much lower magnitude in comparison to the benchmark portfolios).

To further illustrate the results, in Figure 3.1 we plot the evolution of the number of VaR violations and the daily capital charge for the MCR (dashed line) and Min-VaR (solid line) portfolios when the RM approach is used for the conditional covariance matrix. We also plot a horizontal line indicating the threshold value for the "red zone" (9 VaR exceedances in the previous 250 trading days). These graphs clearly show that, for the Global portfolio, the number of VaR violations and the DCR for the MCR portfolio remain well below the ones obtained for the Min-VaR portfolio except for a short period at the beginning of the out-of-sample period. For the other two data sets, the Figure shows that while the DCRs are higher for the MCR portfolios, the number of VaR violations are much lower and - equally important - remained below the "red zone" threshold during the complete out-of-sample period.

Table 3.2 reports the performance of each portfolio strategy in terms of average gross returns, standard deviation of returns, Sharpe ratio, turnover, and break even transaction costs. Returns, standard deviation and Sharpe ratios are annualized. Break even transaction costs are reported in basis points (bp) and returns are reported in percentages. The results for the Global portfolio in Panel A show that the MCR portfolio outperforms the Min-VaR benchmark in terms of gross returns in two out of three specifications. It is also worth noting that, except for the $1/N$ portfolio, the average gross returns fall short of the annualized target return of 10% in all specifications. Nevertheless, the portfolio standard deviation and the turnover are lower for the MCR portfolios than for the Min-VaR portfolios. The SR of the MCR portfolios is significantly higher than that of the Min-VaR portfolios when the LW estimator is used, and the break even transaction cost associated to the MCR portfolios is higher than the one associated to Min-VaR portfolios. As expected, the lowest turnover is achieved by the 1/N portfolio since changes in portfolios compositions are solely due to changes in asset prices. Finally, that for this data set the best performance in terms of returns, turnover and break even transaction costs is achieved by the 1/N portfolio. This result corroborates previous findings in the literature, such as DeMiguel et al. (2009) regarding the outperformance of the 1/N portfolio vis-à-vis more sophisticated portfolio strategies.

Panel B of Table 3.2 reports the results for the 48 Industry portfolios. The results are largely in favor of the MCR portfolio strategy. First, the MCR portfolios deliver gross higher than the Min-VaR and 1/N portfolios in all specifications, and also higher than the annualized target return of 10%. Second, the risk-adjusted performance of the MCR portfolios is significantly higher than that of the Min-VaR portfolio when the LW estimator was used (SR of 1.43 vs. 0.33). Third, the turnover for the MCR portfolio is lower than for the Min-VaR portfolio in all situations. Fourth, the break even transaction costs associated to the MCR portfolio is much higher that the one associated to the Min-VaR portfolio in all specifications. Therefore, we conclude that for this data set the performance of the proposed portfolio policy was highly superior in comparison to that of all benchmark portfolios.

Finally, Panel C of Table 3.2 reports the results for the S&P100 stocks. The performance of the MCR portfolios in terms of gross is again superior to that of Min-VaR portfolio in most of the specifications. The best performance according to this indicator is achieved when the RM model is used. The annualized average gross return obtained with the MCR portfolio is 11.53%, whereas the corresponding number for the Min-VaR portfolio is 2.66%. Moreover, turnover for the MCR portfolios is much lower than for the Min-VaR portfolio in all specifications. Finally, although the risk-adjusted performance of MCR portfolios is not statistically superior than that of Min-VaR portfolios, the break even transaction costs of the former is indeed much higher than that of the later in all situations.

Summarizing the results in Tables 3.1 and 3.2, the optimal MCR portfolios outperform the benchmark portfolios in several aspects. First, the MCR portfolios achieve a better balance between capital requirement levels and the number of VaR violations in comparison to the benchmark portfolios. The average number of VaR violations under the MCR portfolio strategy is the lowest in the vast majority of the specifications for the three data sets. Second, and in contrast to the competing portfolio strategies, the maximum number of VaR violations for the MCR portfolio almost never does not exceed the “yellow zone” upper bound. Third, the turnover for the MCR portfolio is lower than for the Min-VaR portfolio in all specifications. Finally, the MCR portfolios achieve a better performance in terms of gross returns and break even transaction costs in comparison to the Min-VaR benchmark.

Another interesting conclusion that arises from our results is concerned with the performance of the 1/N portfolio. Previous studies found that this portfolio strategy outperforms several sophisticated portfolio strategies in terms of risk adjusted performance and transaction costs. We corroborate this finding, as in some situations the 1/N portfolio outperformed the competing strategies in terms of returns and transaction costs. From the risk management standpoint, however, the performance of the 1/N portfolio may be not so promising. For instance, in the Global portfolio this portfolio strategy delivers higher average and maximum number of VaR violations in comparison to the MCR and Min-VaR portfolios, leading to a higher fraction os days within the “red zone”.

3.3.6 Robustness checks

A potential criticism to our results presented above is that they could be driven by a specific choice of target returns or portfolio re-balancing frequency, or by a specific choice of the econometric specification for the expected returns and the conditional covariance matrix. In order to rule out this possibility, in this section we perform an extensive sensitivity analysis to check the robustness of the MCR portfolio's performance to changes in each of those settings.

Alternative target returns

Table 3.3 reports the average performance indicators (mean DCR, mean Hit, max Hit, fraction of days in "red zone" and in "green zone", gross returns, standard deviation, turnover, Sharpe ratio, and break even transaction costs) across alternative specifications for the covariance matrix for the MCR and Min-VaR portfolios with daily target returns of 2, 4 and 6 bp (equivalent to annual target returns of 5, 10 and 15%). The results are reassuring. First, the average number of VaR exceedances for the MCR portfolio remains lower than for the Min-VaR portfolio in all cases. Second, the performance in terms of gross returns is also better for the MCR portfolios in comparison to the Min-VaR portfolios. Third, MCR portfolios exhibit lower turnover and higher break even transaction costs than Min-VaR portfolios in all cases. Finally, the SR obtained for the MCR portfolios was higher than the one of Min-VaR portfolio in all cases.

It is also worth pointing out important practical consequences for portfolio managers and investors. First, we find that in two cases (Global portfolio and Industry portfolio) an increase in the target return is associated to an increase in average gross returns. However, this increase in gross returns is also accompanied by an increase in standard deviations. Therefore, the benefits of higher target returns is rather unclear from the point of view of risk-adjusted performance. Second, in all situations the turnover increased in response to an increase in target return. This result is expected since higher target returns tend to increase trading activity, thus increasing turnover. Finally, the break even transaction cost usually decline as a responde to an increase in target returns. This suggests that increasing target returns can possibly deteriorate net portfolio returns.

Alternative re-balancing frequencies

The results discussed in the previous subsection are based on the assumption that the investor adjusts her portfolio on a daily basis. The transaction costs incurred with such frequent trading activity can possibly deteriorate the net portfolio performance, as also shown in Table 3.3. Obviously this effect can be avoided by adjusting the portfolio less frequently, such as on a weekly or monthly basis, which in fact is done in practice by many institutional investors. A drawback of rebalancing the portfolio less frequently is that portfolio weights become outdated, which may harm the performance.

The performance of the MCR and Min-VaR portfolio strategies under daily, weekly and monthly re-balancing frequencies are summarized in Table 3.4, showing average performance indicators across all specifications for the covariance matrix. We find that the MCR portfolios outperform the benchmark for all re-balancing frequencies, due to the fact that it achieves a better balance between capital requirement levels and the number of VaR violations. For the Global portfolio, in fact, the MCR portfolios achieve lower DCR levels and a lower number of VaR violations at all re-balancing frequencies. For the other two data sets, the MCR portfolios experience a much lower average number of VaR violations, albeit average capital charges are higher. Furthermore, the MCR portfolio exhibit lower portfolio turnover, higher gross returns, higher Sharpe ratios, and higher break even transaction cost in the vast majority of the cases.

As expected, we find that lowering the re-balancing frequency results in a substantial reduction in portfolio turnover. This decline is more pronounced for the MCR portfolios than for the Min-VaR portfolios. The lower portfolio turnover contributes to generate higher break even transaction cost for the MCR portfolio in all cases.

Alternative model for the expected returns

The VAR(1) model in (3.9) contains $N + N^2$ unknown coefficients. Its use for generating expected returns thus entails a large amount of estimation uncertainty for the values of N considered here. As a more parsimonious alternative we consider using univariate AR(1) models for each individual asset, i.e. the matrix B in (3.9) is restricted to be diagonal. A drawback of this simplification is that it ignores possible important cross-correlations among the assets in the portfolio.

The results in Table 3.5 indicate that regardless of the specification used for the expected returns, the MCR portfolios perform better in comparison to the benchmark portfolios. Compared to the results based on the unrestricted VAR(1) model in Table 3.1, we observe that for the Global portfolio the average DCR and the average number of violations tends to be higher when univariate AR(1) models are used. However, the opposite conclusion is reached for the portfolio of S&P100 stocks. Finally, we find that that modeling expected returns with a AR(1) results in higher SR and higher break even transaction costs compared to those originally obtained with a VAR(1) model.

Alternative model for the conditional covariance matrix

Table 3.6 report the results obtained when the conditional covariance matrix is described by means of the constant conditional correlation (CCC) model of Bollerslev (1990). As its name suggests, in this model conditional correlations are assumed to be constant rather than time-varying as in the DCC model, that is, $P_t = P$ in (1.11) where P is the unconditional correlation matrix of the standardized returns. The results indicate that the MCR portfolios still outperform the benchmarks when conditional correlations are assumed to be constant.

3.4 Concluding Remarks

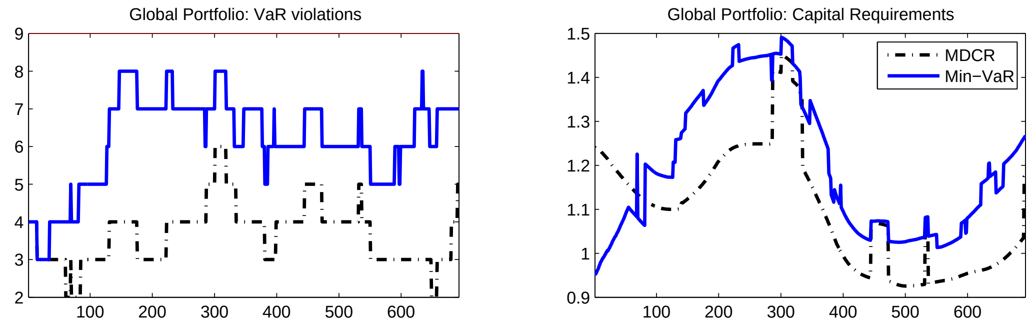
Previous empirical studies have found that banks and other large financial institutions tend to overestimate the VaR of their asset portfolios. This results in prohibitive amounts of regulatory capital requirements (thus generating opportunity costs), while it also gives rise to reputational concerns. On the other hand, it also is not attractive for banks to underestimate their risk levels as this may lead to an excessive number of VaR violations and higher-than-expected losses. In addition, the regulations in the Basel II Accord impose a penalty on the regulatory capital in case VaR exceedances occur too frequently, such that lower VaR estimates may actually increase capital requirements.

In this paper we proposed a novel approach based on active portfolio selection that alleviates these problems. The methodology involves setting portfolio weights in order to minimize the level of capital requirements, subject to a restriction on the number of VaR exceedances and other constraints (involving the target performance of the portfolio, for example).

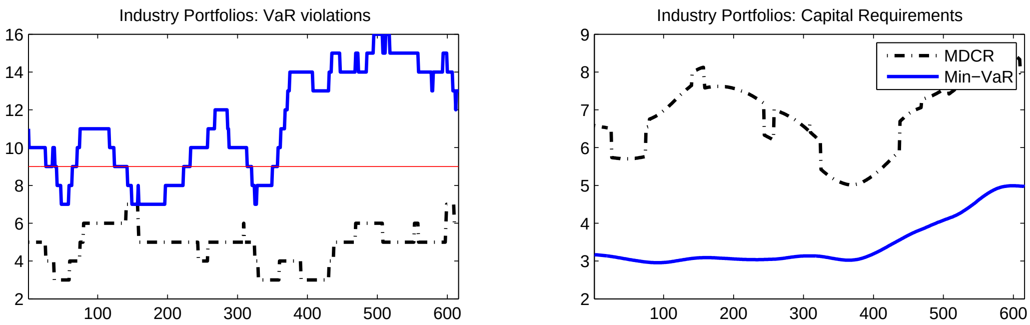
An empirical application to three portfolios composed of different types of assets demonstrated that the developed approach is able to provide a much better balance between capital requirement levels and the number of VaR violations compared to the minimum-VaR portfolio and the 1/N portfolio. This result is robust to the specification of the conditional covariance matrix, to the specification of the expected returns, to the level of target returns, and to the portfolio re-balancing frequency.

Table 3.1: Daily capital requirements and the number of VaR violations.
The Table reports the average daily capital requirement (Mean DCR), the average and maximum number of VaR violations (Mean Hit and Max Hit, respectively), and the fraction of days the number of VaR violations are within the "green zone" (i.e. below 5) and within the "red zone" (i.e. above 9). The numbers are based on subsequent (rolling) periods of 250 out-of-sample observations. One, two, and three asterisks indicate that the statistic is significantly lower than that of the Min-VaR portfolio at the 10%, 5%, and 1% level, respectively. Capital requirements are measured in percentages.

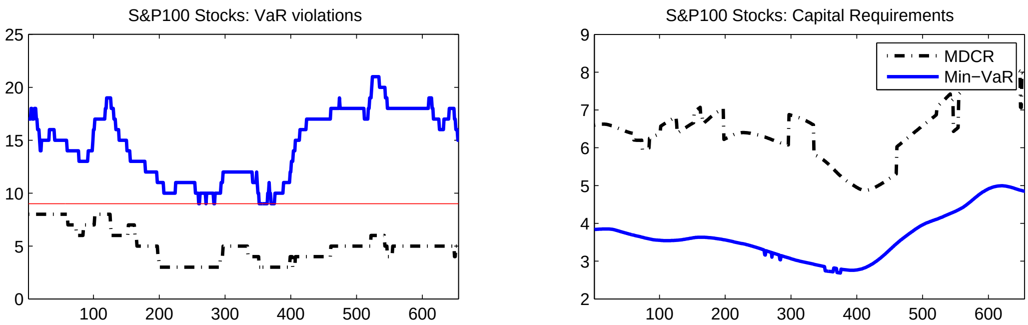
	Mean DCR (%)	Mean Hit	Max Hit	% of days in red zone	% of days in green zone
Panel A: Global Portfolio					
Covariance model: DCC					
MCR	1.20***	2.60	5	0	97.35
Min-VaR	1.23	2.46	5	0	93.75
1/N	2.93	6.07	9	0	44.81
Covariance model: Risk Metrics					
MCR	1.10***	3.65***	6	0	91.00
Min-VaR	1.20	6.16	8	0	34.96
1/N	3.13	5.89	10	0.64	44.07
Covariance model: Ledoit-Wolf					
MCR	1.41***	1.18***	6	0	96.08
Min-VaR	1.49	1.52	6	0	96.08
1/N	3.07	7.00	16	19.49	46.61
Panel B: Industry Portfolios					
Covariance model: DCC					
MCR	6.65	3.59***	6	0	85.93
Min-VaR	3.71	9.13	17	28.37	37.37
1/N	4.90	8.95***	15	26.87	36.91
Covariance model: Risk Metrics					
MCR	6.83	4.85***	7	0	49.13
Min-VaR	3.46	11.26	16	48.67	28.84
1/N	5.05	7.04***	12	21.45	42.33
Covariance model: Ledoit-Wolf					
MCR	6.81	3.50***	10	6.57	78.43
Min-VaR	3.60	10.66	25	28.84	42.21
1/N	4.58	8.44***	24	22.61	63.32
Panel C: S&P 100 Stocks					
Covariance model: DCC					
MCR	7.00	4.28***	9	0	70.20
Min-VaR	3.74	8.12	17	27.15	56.51
1/N	4.39	7.29***	18	26.82	68.76
Covariance model: Risk Metrics					
MCR	6.51	5.02***	8	0	53.53
Min-VaR	3.65	14.64	21	69.76	27.59
1/N	4.62	6.40***	10	9.71	42.38
Covariance model: Ledoit-Wolf					
MCR	6.06	4.02***	12	18.54	72.85
Min-VaR	4.14	7.44	20	27.15	68.98
1/N	4.98	9.02	28	26.82	71.52



(c) Global portfolio



(d) Industry portfolios



(e) S&P 100 stocks

Figure 3.1: Number of VaR violations (left) and capital requirements (right) for the MCR (dashed line) and Min-VaR (solid line) portfolios when the Riskmetrics specification is used to model the conditional covariance matrix. The horizontal line indicates the “red zone” threshold (9 violations in the previous 250 trading days).

Table 3.2: Portfolio performance

The Table reports the average gross portfolio return, the standard deviation of portfolio returns, portfolio turnover, the Sharpe ratio, and the break even transaction cost. Returns, standard deviation and Sharpe ratio are annualized. The numbers are based on out-of-sample observations. One, two, and three asterisks indicate that the Sharpe ratio is significantly higher than that of the Min-VaR portfolio at the 10%, 5% and 1% levels, respectively. The annualized target return is 10%. Returns are reported in percentages and break even costs are reported in basis points.

	Gross returns (%)	Std. dev.	Sharpe ratio	Turnover	Break even cost (bp)
Panel A: Global Portfolio					
Covariance model: DCC					
MCR	4.56	3.59	1.27	0.46	3.88
Min-VaR	4.23	3.81	1.11	0.69	2.40
Covariance model: Risk Metrics					
MCR	4.32	3.56	1.21	0.49	3.49
Min-VaR	5.02	3.96	1.27	0.79	2.49
Covariance model: Ledoit-Wolf					
MCR	3.47	3.82	0.91^{**}	0.44	3.08
Min-VaR	0.34	4.99	0.07	0.68	0.20
1/N	11.45	8.77	1.31	0.01	704.72
Panel B: Industry Portfolios					
Covariance model: DCC					
MCR	14.42	19.36	0.75	0.41	13.23
Min-VaR	4.81	10.82	0.44	0.99	1.91
Covariance model: Risk Metrics					
MCR	10.13	19.63	0.52	0.54	7.25
Min-VaR	6.79	10.52	0.65	1.04	2.55
Covariance model: Ledoit-Wolf					
MCR	26.91	18.80	1.43^{**}	0.37	26.33
Min-VaR	3.68	11.31	0.33	0.80	1.82
1/N	6.35	14.32	0.44	0.01	476.05
Panel C: S&P 100 Stocks					
Covariance model: DCC					
MCR	3.63	20.27	0.18	0.54	2.65
Min-VaR	2.84	11.65	0.24	1.53	0.73
Covariance model: Risk Metrics					
MCR	11.53	19.55	0.59	0.61	7.28
Min-VaR	2.66	12.52	0.21	1.52	0.69
Covariance model: Ledoit-Wolf					
MCR	3.57	16.98	0.21	0.61	2.31
Min-VaR	5.07	13.06	0.39	1.40	1.42
1/N	3.32	13.66	0.24	0.01	162.19

Table 3.3: Robustness checks: alternative target returns

The Table reports for each target return (2 bp, 4 bp, and 6 bp) the average performance indicators across alternative specifications for the covariance matrix.

	Mean DCR (%)	Mean Hit	Max Hit	% of days in red zone	% of days in green zone	Gross return (%)	Std. dev.	Sharpe ratio	Turnover	break even cost (bp)
Panel A: Global Portfolio										
MCR portfolios										
2 bp	1.20	3.03	6.67	0	83.3	4.08	3.54	1.16	0.31	5.18
4 bp	1.23	2.48	5.67	0	94.8	4.11	3.66	1.13	0.46	3.49
6 bp	1.31	2.56	5.67	0	96.6	4.54	3.90	1.17	0.63	2.82
Min-VaR portfolios										
2 bp	1.28	3.71	6.67	0	72.5	3.10	4.18	0.81	0.65	1.82
4 bp	1.31	3.38	6.33	0	74.9	3.20	4.25	0.82	0.72	1.70
6 bp	1.40	3.41	6.67	0	73.8	3.34	4.44	0.81	0.82	1.57
Panel B: Industry Portfolios										
MCR portfolios										
2 bp	6.85	4.06	8.00	3.65	72.66	16.86	19.72	0.86	0.41	16.85
4 bp	6.76	3.98	7.67	2.19	71.16	17.15	19.26	0.90	0.44	15.60
6 bp	6.72	3.54	7.33	2.42	84.47	17.87	20.04	0.91	0.43	16.15
Min-VaR portfolios										
2 bp	3.35	10.44	19.33	35.52	36.14	4.01	11.04	0.37	0.93	1.67
4 bp	3.59	10.35	19.33	35.29	36.14	5.09	10.88	0.47	0.94	2.09
6 bp	3.39	10.35	19.33	35.29	36.14	5.24	10.92	0.48	0.97	2.10
Panel C: S&P100 Stocks										
MCR portfolios										
2 bp	6.50	4.31	9.33	5.30	68.47	6.43	18.78	0.32	0.57	4.40
4 bp	6.52	4.44	9.67	6.18	65.53	6.24	18.93	0.33	0.58	4.08
6 bp	6.36	4.33	9.33	6.88	71.19	4.37	19.05	0.23	0.61	2.73
Min-VaR portfolios										
2 bp	3.84	9.94	19.00	40.88	51.03	3.55	12.41	0.28	1.48	0.96
4 bp	3.84	10.07	19.33	41.35	51.03	3.52	12.41	0.28	1.48	0.95
6 bp	3.85	10.07	19.33	41.35	51.03	3.48	12.42	0.28	1.49	0.94

Table 3.4: Robustness checks: alternative re-balancing frequencies

The Table reports for each re-balancing frequency (daily, weekly, and monthly) the average performance indicators across alternative specifications for the covariance matrix.

	Mean DCR (%)	Mean Hit	Max Hit	% of days in red zone	% of days in green zone	Gross return (%)	Std. dev.	Sharpe ratio	Turnover	break even cost (bp)
Panel A: Global Portfolio										
MCR portfolios										
Daily	1.23	1.82	5.67	0	94.81	4.11	3.66	1.13	0.46	3.49
Weekly	1.30	1.53	5.00	0	95.90	6.35	4.61	1.38	0.10	24.10
Monthly	1.26	2.55	6.33	0	82.34	3.92	3.74	1.05	0.02	63.96
Min-VaR portfolios										
Daily	1.31	2.48	6.33	0	74.93	3.20	4.25	0.82	0.72	1.70
Weekly	1.35	3.17	7.67	0	65.25	3.42	4.18	0.82	0.57	2.41
Monthly	1.37	3.20	8.33	0	61.86	4.87	4.18	1.18	0.69	2.80
Panel B: Industry Portfolios										
MCR portfolios										
Daily	6.76	2.83	7.67	0.04	71.16	17.15	19.26	0.90	0.44	15.60
Weekly	6.96	3.30	9.33	0.04	62.05	20.03	19.34	1.04	0.15	51.56
Monthly	6.83	2.72	8.00	0.04	80.08	18.54	19.69	0.95	0.07	100.65
Min-VaR portfolios										
Daily	3.59	7.37	19.33	9.53	36.14	5.09	10.88	0.47	0.94	2.09
Weekly	3.47	5.38	13.33	7.61	44.14	5.26	10.30	0.51	0.76	2.75
Monthly	3.52	5.79	14.33	7.61	44.02	6.17	10.21	0.61	0.90	2.70
Panel C: S&P100 Stocks										
MCR portfolios										
Daily	6.52	3.22	9.67	0.04	65.53	6.24	18.93	0.33	0.58	4.08
Weekly	6.52	3.36	10.33	0.04	68.43	7.60	18.65	0.40	0.24	13.19
Monthly	6.76	3.83	11.33	0.04	55.48	3.72	19.04	0.19	0.15	14.43
Min-VaR portfolios										
Daily	3.84	7.29	19.33	9.12	51.03	3.52	12.41	0.28	1.48	0.95
Weekly	3.91	6.26	16.67	6.95	53.83	1.27	11.76	0.11	1.20	0.41
Monthly	3.93	6.36	17.33	8.09	52.32	4.06	11.42	0.36	1.41	1.13

Table 3.5: Robustness checks: alternative model for the expected returns

The Table reports the performance indicators when expected returns are described according to a AR(1) model for each of the individual assets in the data set. One, two, and three asterisks indicate that the portfolio policy outperformed the Min-VaR portfolio at the 10%, 5% and 1% levels, respectively.

	Mean DCR (%)	Mean Hit	Max Hit	% of days in red zone	% of days in green zone	Gross return (%)	Std. dev.	Sharpe ratio	Turnover	break even cost (bp)
Panel A: Global Portfolio										
Covariance model: DCC										
MCR	1.55	4.66	7	0	97.4	7.63	4.48	1.70	0.68	4.35
Min-VaR	1.41	4.02	6	0	72.7	7.88	4.39	1.80	0.70	4.34
1/N	2.88	5.35	9	0	46.9	11.45	8.77	1.31	0.01	704.72
Covariance model: Risk Metrics										
MCR	1.40	3.84***	7	0	69.3	7.13	4.40	1.62	0.71	3.92
Min-VaR	1.40	8.72	13	27.33	26.5	6.69	4.48	1.49	0.73	3.56
1/N	3.04	5.35***	9	0	62.9	11.45	8.77	1.31	0.01	704.72
Covariance model: Ledoit-Wolf										
MCR	1.60	2.70***	9	0	89.9	6.44	4.83	1.33	0.61	4.13
Min-VaR	1.59	3.78	10	0.11	75.5	6.60	4.73	1.40	0.64	4.03
1/N	2.90	5.67	15	15.04	68.6	11.45	8.77	1.31	0.01	704.72
Panel B: Industry Portfolios										
Covariance model: DCC										
MCR	6.90	2.70***	5	0	85.93	14.20	21.03	0.68	0.14	39.50
Min-VaR	3.18	9.39	17	28.84	37.49	4.44	9.92	0.45	0.32	5.50
1/N	4.80	8.43***	15	22.61	34.49	6.35	14.32	0.44	0.01	476.05
Covariance model: Risk Metrics										
MCR	6.75	4.75***	7	0	61.94	12.21	19.90	0.61	0.23	20.67
Min-VaR	3.17	12.22	17	65.74	28.84	8.16	9.82	0.83	0.34	9.39
1/N	4.85	6.12***	10	5.31	46.14	6.35	14.32	0.44	0.01	476.05
Covariance model: Ledoit-Wolf										
MCR	6.81	3.18***	10	0	78.43	32.16	19.44	1.65***	0.24	47.73
Min-VaR	3.06	10.74	24	28.84	44.87	3.00	10.70	0.28	0.17	6.90
1/N	4.83	10.18	25	26.87	39.22	6.35	14.32	0.44	0.01	476.05
Panel C: S&P 100 Stocks										
Covariance model: DCC										
MCR	6.46	4.59***	10	5.41	70.20	5.53	19.04	0.29	0.27	8.15
Min-VaR	3.08	9.29	21	28.15	51.55	4.92	10.22	0.48	0.54	3.58
1/N	4.35	6.69***	18	21.63	71.74	3.32	13.66	0.24	0.01	162.19
Covariance model: Risk Metrics										
MCR	6.34	4.85***	8	0	64.90	11.35	19.64	0.58	0.33	13.30
Min-VaR	2.85	17.89	24	72.41	27.59	4.53	10.63	0.43	0.56	3.20
1/N	4.55	6.59***	13	22.41	60.26	3.32	13.66	0.24	0.01	162.19
Covariance model: Ledoit-Wolf										
MCR	5.93	4.39***	13	15.12	71.74	4.18	16.02	0.26	0.36	4.55
Min-VaR	3.66	8.13	24	28.15	67.99	3.75	10.56	0.36	0.47	3.14
1/N	4.97	7.05***	24	25.72	71.74	3.32	13.66	0.24	0.01	162.19

Table 3.6: Robustness checks: alternative model for the conditional covariance matrix
The Table reports the performance indicators when conditional covariances are described according to the constant conditional correlation (CCC) model.

	Mean DCR (%)	Mean Hit	Max Hit	% of days in red zone	% of days in green zone	Gross return (%)	Std. dev.	Sharpe ratio	Turnover	break even cost (bp)
Panel A: Global Portfolio										
MCR	1.24	3.32	7	0	83.58	4.65	3.60	1.29	0.46	3.95
Min-VaR	1.23	2.46	5	0	93.75	4.17	3.84	1.09	0.69	2.37
1/N	2.92	6.07	9	0	44.81	11.45	8.77	1.31	0.01	704.72
Panel B: Industry Portfolios										
MCR	6.82	4.38***	6	0	65.86	13.38	19.62	0.68	0.41	12.49
Min-VaR	3.70	9.13	17	28.37	37.37	4.79	10.83	0.44	0.99	1.90
1/N	4.90	8.95	15	26.87	36.91	6.35	14.32	0.44	0.01	476.05
Panel C: S&P 100 Stocks										
MCR	6.99	4.09***	9	0	70.20	9.07	19.84	0.46	0.54	6.54
Min-VaR	3.74	8.12	17	27.15	56.51	2.78	11.66	0.24	1.53	0.72
1/N	4.40	7.29***	18	26.82	68.76	3.32	13.66	0.24	0.01	162.19

Chapter 4

Conclusions and directions for future research

The application of multivariate volatility models in market risk modeling is undoubtedly one the most important frontiers in financial econometrics. From the regulatory point of view, multivariate volatility models play a major role as they are very useful to measure and manage financial risks, and to analyze important issues such as financial contagion, hedging and portfolio diversification. Moreover, from the econometric perspective, multivariate models poses a number of challenging issues ranging from model specification, estimation and testing. In this sense, the study of this class of models is of main concern for both researchers and practitioners in the financial industry.

In this thesis, we study the application of multivariate volatility models in both risk measurement and management. In the first part of the thesis, we compare the performance of multivariate vs. univariate volatility models in forecasting the value-at-risk (VaR) of a given portfolio of assets. Existing literature has tried to answer this question by analyzing only small portfolios and using a testing framework not appropriate for ranking VaR models. We, on the other hand, provide a more comprehensive look at the problem of portfolio VaR forecasting by using more appropriate statistical tests of comparative predictive ability. Moreover, we compare univariate vs. multivariate VaR models in the context of diversified portfolios containing a large number of assets and also provide evidence based on Monte Carlo experiments. In the third chapter we shift our attention to an active risk management problem and propose a novel optimization problem based on the Basel II capital requirement formula to obtain optimal portfolios with minimum capital requirements subject to a given number of violations over the previous trading year.

The main contributions of this thesis can be summarized as follows:

- We find that multivariate volatility models outperform univariate counterparts in forecasting portfolio VaR in a controlled Monte Carlo experiment. The results indicate that multivariate models tend to be chosen more often, and that the mean squared error (MSE) associated to the VaR forecasts are lower in comparison to univariate models. Moreover, this result is robust to the parametrization of the simulated model.

- We also compare the comparative performance of multivariate models when applied to real market data. For that purpose, we consider a more appropriate testing framework for ranking VaR models based on the conditional predictive ability test of Giacomini and White (2006). Moreover, we follow a different strategy of previous studies and consider large and diversified portfolios constructed from three sets of different type of assets. The results are still in favor of multivariate models, which confirms the hypothesis that these models are very useful for risk measurement purposes.

- We propose a novel approach to active risk management based on the Basel II regulations to obtain optimal portfolios with minimum capital requirements. The novel methodology - named MCR portfolios - help overcome the drawbacks of both over- and understatement of a bank's VaR as discussed above. Specifically, we propose to determine optimal portfolio weights by directly minimizing the daily capital requirements, but subject to a restriction on the number of VaR violations during the preceding year. Implicitly, our approach aims to find the optimal balance between the level of VaR measures and the number of VaR violations, thus leading to the lowest possible level of capital requirements.

- We apply our proposed portfolio selection policy to large and diversified portfolios constructed from three sets of different type of assets, and compare its performance to various benchmark portfolios. We find a lower average number of VaR violations for the MCR portfolios. In addition for the MCR portfolios the number of VaR violations does not enter the "red zone" in the vast majority of the specifications. This is in sharp contrast to the benchmark portfolios, for which we frequently find more than ten VaR violations over the preceding trading year. Finally, the performance of the MCR portfolios in terms of gross returns, risk-adjusted returns, portfolio turnover, and breakeven transaction costs are superior to those of minimum-VaR portfolios. Altogether, these results strongly suggests that multivariate models are also a powerful tool to risk management purposes.

Finally, it is also important to emphasize that the literature in multivariate models can be considered in its early stages. Up to this moment, one of the main difficulties is concerned with the application of multivariate models in very large dimensional problems. One can argue that large dimensional problems are precisely the ones in which multivariate models are most needed, for obvious reasons. Therefore, we can highlight some promising lines for future research regarding those issues:

- ***Reduction of estimation error in large dimensional problems***: One of the main difficulties in applying parametric multivariate models to problems involving a large number of assets (with more than 100 assets, for instance) is the estimation error. Most of the existing econometric specifications are still highly parameterized, which entails some degree of estimation error in the parameters and also in the estimated covariance matrices. Therefore, a promising research line is the study and development of alternative econometric specifications for alleviating the problem of estimation error. One of the seminal approaches in this line is the shrinkage estimators proposed by Ledoit and Wolf (2003), Ledoit and Wolf (2004a), and Ledoit and Wolf (2004b). The advantages of their approach is that it can be applied to very large problems and is computationally fast. Moreover, the authors show that the resulting covariance matrix obtained by applying the shrinkage methodology contains less estimation error than the traditional sample covariance matrix. One of the deficiencies in their approach, however, is that the proposed shrinkage estimators are based on an unconditional covariance matrix. Therefore, one potential extension is to consider a *conditional* version of the Ledoit-Wolf class of shrinkage estimators. Another useful approach for alleviating the problem of estimation errors are factor models. These models are based on statistical techniques of dimensionality-reduction, such as principal components analysis, and usually offer more parsimonious specifications, which facilitates the estimation procedure and entails a lower degree of estimation error. In this sense, it is worth extending the factor models proposed by Aguilar and West (2000), Alexander (2001), and Han (2006), among others.

- ***Efficient estimation in large dimensional problems***: Another important difficulty is concerned with the estimation methods for multivariate volatility models. The methods currently available are still very computational demanding and possibly limited to problems involving a given number of assets. The two-step estimation procedure for dynamic conditional correlation models can be seen as an important improvement in this direction. However, as Engle et al. (2008) point out, the two-step procedure fails to estimate parameters when very large systems are considered. The authors propose an alternative procedure able to estimate the parameters of dynamic conditional correlation models for very large systems of assets in a reasonable computational time. Therefore, a promising research line is the study of alternative estimation methods for multivariate volatility models that enables the user to i) work with portfolios with a large number of assets and ii) obtain consistent parameter estimates in a reasonable computational time.

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